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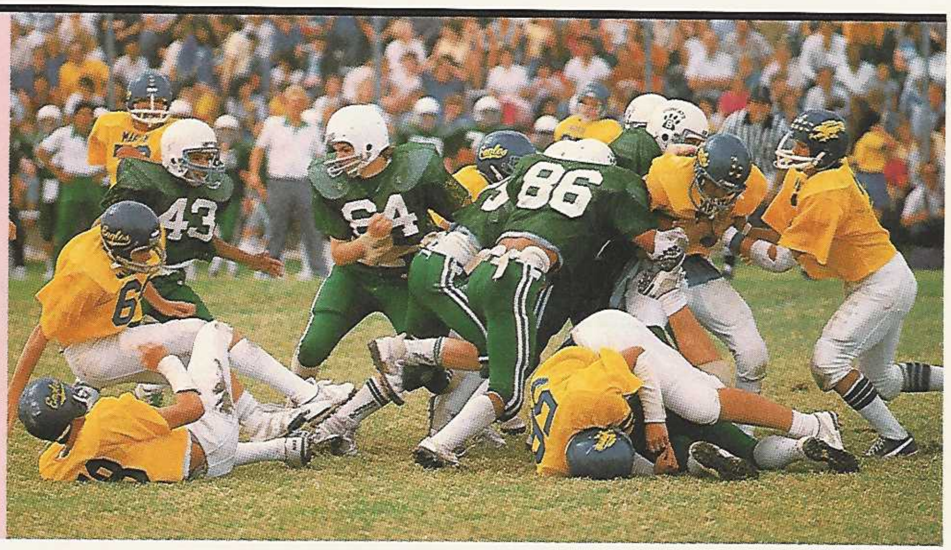
CHAPTER

7 Linear Equations in Two Variables

Tickets to a football game at Smith High School cost \$1.25 for students and \$3.00 for adults. The total receipts for a game with University High School were \$1,020. Write an equation using x for the number of students and y for the number of adults attending the game.



7–1 ■ Ordered pairs and the rectangular coordinate system

Linear equations in two variables

In chapter 2, we developed methods for solving linear equations (first-degree equations) in one variable. All such equations could be stated in the form

$$ax + b = 0$$

where a and b are real number constants and $a \neq 0$. In this chapter, we expand our work to linear equations in *two variables*. The equations

$$3x + 4y = 8 \quad \text{and} \quad 4y - x = 0$$

are examples of linear equations in two variables.

Definition

A **linear equation in two variables** x and y is any equation that can be written in the form

$$ax + by = c$$

where a, b, and c are real numbers and a and b are not both zero.

Our primary concern with all equations is finding their **solution(s).** These are the replacement values for the variable(s) that satisfy the equation. In an equation in two variables, x and y, any *pair* of values for x and y that satisfies the equation is a solution of the equation.

Example 7–1 A

Given the linear equation $3x + 2y = 6$, determine if the given values of x and y are solutions of the equation.

1. Let $x = 2$ and $y = 0$

$$\begin{aligned} 3x + 2y &= 6 \\ 3(2) + 2(0) &= 6 && \text{Replace } x \text{ with 2 and } y \text{ with 0 in the equation} \\ 6 + 0 &= 6 && \text{Multiply as indicated} \\ 6 &= 6 && \text{(True)} \end{aligned}$$

We see that the given values satisfy the equation, so $x = 2$ and $y = 0$ is a solution.

2. Let $x = 1$ and $y = \frac{3}{2}$

$$\begin{aligned} 3x + 2y &= 6 \\ 3(1) + 2\left(\frac{3}{2}\right) &= 6 && \text{Replace } x \text{ with 1 and } y \text{ with } \frac{3}{2} \text{ in the equation} \\ 3 + 3 &= 6 \\ 6 &= 6 && \text{(True)} \end{aligned}$$

Again we see that the values $x = 1$ and $y = \frac{3}{2}$ satisfy the equation and thereby form a solution.

3. Let $x = 3$ and $y = 1$

$$\begin{aligned} 3x + 2y &= 6 \\ 3(3) + 2(1) &= 6 && \text{Replace } x \text{ with 3 and } y \text{ with 1 in the equation} \\ 9 + 2 &= 6 \\ 11 &= 6 && \text{(False)} \end{aligned}$$

We conclude that $x = 3$ and $y = 1$ *do not* satisfy the equation and hence *do not form a solution.*

▶ ***Quick check*** Determine if $x = 1$ and $y = -1$ and if $x = 2$ and $y = 0$ are solutions for $3x + y = 2$

Ordered pairs of numbers

The pairs of values for x and y used in example 7–1 A may be written as a pair of numbers. We separate them by a comma and place them inside the parentheses. The value of x is *always* given first. That is, the pair of numbers is written (x,y). In the examples, we used the pairs

$$(2,0),\ \left(1,\frac{3}{2}\right),\ \text{and } (3,1)$$

Pairs of numbers written in this special *order* (with the x value always first) are called **ordered pairs of numbers.** The *first number* of the ordered pair (the value of x) is called the *first component* of the ordered pair. The *second number* (the value of y) is called the *second component* of the ordered pair.

To determine ordered pairs that are solutions of an equation in two variables, we use the following procedure:

Finding ordered pair solutions

1. Choose a value for one of the variables.
2. Replace the variable with this known value and solve the resulting equation for the other variable.

■ Example 7–1 B

Using the given value of one of the variables, find the remaining variable. Write the ordered pair solution of the equation.

1. Given $y = 2x + 1$

a. Let $x = 3$

$y = 2x + 1$
$y = 2(3) + 1$ Replace x with 3 in the equation
$y = 6 + 1$ Multiply as indicated
$y = 7$ Add in right member

The ordered pair (3,7) is a solution.

b. Let $x = -2$.

$y = 2x + 1$
$y = 2(-2) + 1$ Replace x with -2 in the equation
$y = -4 + 1$ Solve for y
$y = -3$

The ordered pair $(-2,-3)$ is a solution.

c. Let $y = 5$

$y = 2x + 1$
$(5) = 2x + 1$ Replace y with 5 in the equation
$4 = 2x$ Solve for x
$2 = x$

The ordered pair (2,5) is a solution.

Note We can choose *infinitely many* values for x and get a corresponding value of y for each one. That means there are infinitely many solutions.

2. Given $3x - 2y = 4$

a. Let $x = 2$

$3x - 2y = 4$
$3(2) - 2y = 4$ Replace x with 2 in the equation
$6 - 2y = 4$ Solve for y
$-2y = -2$
$y = 1$

The ordered pair (2,1) is a solution.

b. Let $y = 7$

$$\begin{aligned} 3x - 2y &= 4 \\ 3x - 2(7) &= 4 && \text{Replace } y \text{ with 7 in the equation} \\ 3x - 14 &= 4 && \text{Solve for } x \\ 3x &= 18 \\ x &= 6 \end{aligned}$$

The ordered pair (6,7) is a solution.

3. Given $y = 6$
We can rewrite this equation as
$y + 0 \cdot x = 6$

a. Let $x = 3$

$$\begin{aligned} y + 0 \cdot x &= 6 \\ y + 0(3) &= 6 && \text{Replace } x \text{ with 3 in the equation} \\ y + 0 &= 6 && \text{Solve for } y \\ y &= 6 \end{aligned}$$

The ordered pair (3,6) is a solution.

b. Let $x = -7$

$$\begin{aligned} y + 0 \cdot x &= 6 \\ y + 0(-7) &= 6 && \text{Replace } x \text{ with } -7 \text{ in the equation} \\ y + 0 &= 6 && \text{Solve for } y \\ y &= 6 \end{aligned}$$

The ordered pair $(-7,6)$ is a solution.

Note No matter what *value* we choose for *x, y* will *always* be 6. Then *every* ordered pair will have a 6 in the second position.

4. Given $x + 3 = 0$
We can rewrite this equation as $x = -3$ and then as
$x + 0 \cdot y = -3$.

a. Let $y = 1$

$$\begin{aligned} x + 0 \cdot y &= -3 \\ x + 0(1) &= -3 && \text{Replace } y \text{ with 1 in the equation} \\ x + 0 &= -3 && \text{Solve for } x \\ x &= -3 \end{aligned}$$

The ordered pair $(-3,1)$ is a solution.

b. Let $y = -4$

$$\begin{aligned} x + 0 \cdot y &= -3 \\ x + 0(-4) &= -3 && \text{Replace } y \text{ with } -4 \text{ in the equation} \\ x + 0 &= -3 && \text{Solve for } x \\ x &= -3 \end{aligned}$$

The ordered pair $(-3,-4)$ is a solution.

Note No matter what *value* we choose for *y, x* will *always* be -3. Then every ordered pair will have a -3 in the first position.

From examples 3 and 4, we can see that solutions of linear equations in two variables that can be written in the form

$$x = a \quad \text{or} \quad y = b$$

where a and b are constants, have very special characteristics. Given

1. $x = a$, the *first component* in every ordered pair is always the number a.
2. $y = b$, the *second component* in every ordered pair is always the number b.

▶ ***Quick check*** In the following equations, find the missing variables value for each variable given.
a. $2x + y = 1$; $x = 2$, $x = -3$
b. $2y - 3x = 1$; $y = -4$, $y = 0$

The rectangular coordinate plane

In chapter 1, we associated the set of real numbers with points on a straight line and called this the number line. This number line was then used to draw the graph of the solution of an equation or inequality in one variable. Now we associate the solutions of a linear equation *in two variables* with *points* on a flat surface, called a *plane*.

We take two number lines, one horizontal and the other vertical, on the plane that are perpendicular to each other, and call them *axes*. These two number lines are represented in figure 7–1. The horizontal line (x-axis) is associated with values of x, and the vertical line (y-axis) is associated with values of y. Together, the x- and y-axes form the **rectangular coordinate plane** (or Cartesian coordinate plane).

Note The invention of this graphing method (relating the algebraic concept of an ordered number pair with the geometric concept of a point in the plane) is attributed to French mathematician and philosopher René Descartes (1596–1650). This combination of algebra and geometry has become known as *analytic geometry* or *coordinate geometry*.

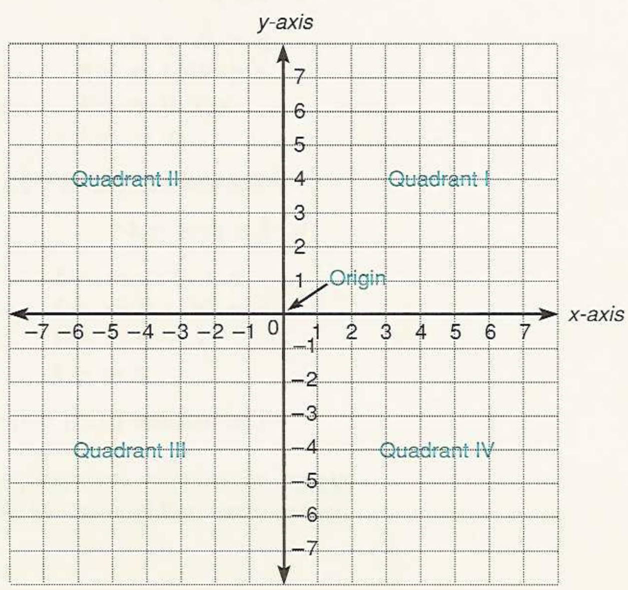


Figure 7–1

The point at which the two axes intersect is their common zero point and is called the *origin.* The origin corresponds to the ordered pair (0,0). The two axes separate the coordinate plane into four regions called *quadrants.* These quadrants are named as shown in figure 7–1. Points that lie on the x-axis or y-axis *do not* lie in any of the quadrants.

On the x-axis, numbers to the right of the origin are positive and those to the left of the origin are negative. On the y-axis, the positive numbers are above the origin, and the negative numbers are below the origin. For each point on the x-axis, $y = 0$, and for each point on the y-axis, $x = 0$.

Note We have chosen to make each unit on the axes equal to 1. Other choices are possible and, in fact, may be necessary in some instances.

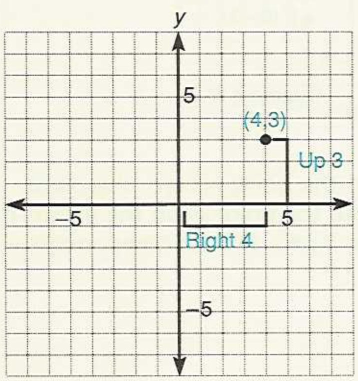


Figure 7–2

Each ordered pair (x,y) corresponds to *exactly* one point, called the *graph* of the ordered pair. To find the location of such a point is called *plotting* the point. We can plot any ordered pair, (x,y), on our coordinate plane if we consider the ordered pair as two instructions to direct us from the origin to the proper location of the point. To plot the point that corresponds to the ordered pair (4,3), we start at the origin. Since the x-value, also called the **abscissa** of the point, is 4, we move four units *to the right* (the positive direction) along the x-axis. From this position, the y-value, also called the **ordinate** of the point, instructs us to move *up* (the positive direction) three units parallel to the y-axis. The abscissa and ordinate are usually called the *coordinates* of a point. We have plotted the point that is the graph of the ordered pair (4,3) as shown in figure 7–2.

Similarly, to plot the graph of $(-3,-5)$, we start at the origin. Since $x = -3$, we move three units *to the left* (the negative direction). Next, $y = -5$ instructs us to move *down* (the negative direction) five units parallel to the y-axis. We have plotted the graph of the ordered pair $(-3,-5)$ as shown in figure 7–3.

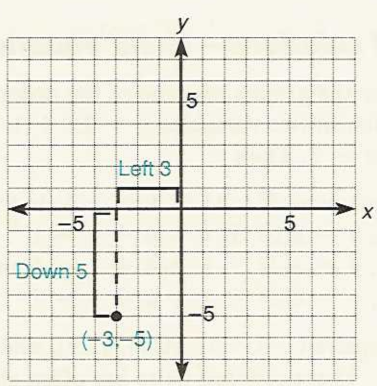


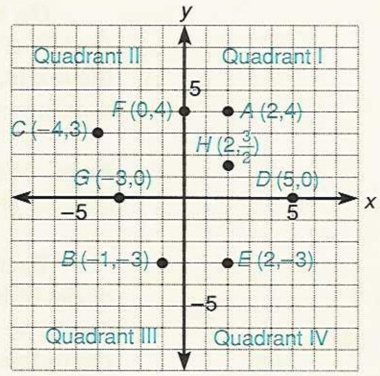
Figure 7–3

Points are always named by capital letters. The notation $A(x,y)$ indicates that the *name* of the point is A and the *coordinates* of the point are (x,y).

Example 7–1 C

Plot the following points.

a. $A(2,4)$
b. $B(-1,-3)$
c. $C(-4,3)$
d. $D(5,0)$
e. $E(2,-3)$
f. $F(0,4)$
g. $G(-3,0)$
h. $H\left(2,\dfrac{3}{2}\right)$



Note Whenever either one of the coordinates of the point is zero, the point is located on an axis. If the first component is zero, the point is on the y-axis. If the second component is zero, the point is on the x-axis. If both components are zero, the point is the origin.

We can see in the diagram that a point is in

1. quadrant I when x and y are both positive,
2. quadrant II when x is negative and y is positive,
3. quadrant III when x and y are both negative,
4. quadrant IV when x is positive and y is negative.

Graphs of solutions—linear equations in two variables

Now consider the graphs of some of the ordered pairs that are solutions of the linear equation in two variables $y = 2x + 1$. Suppose we let $x = -3$, $x = 0$, $x = 2$, and $x = 3$. Then

when $x = -3$, $y = 2(-3) + 1 = -6 + 1 = -5$ — Replace x with -3
when $x = 0$, $y = 2(0) + 1 = 0 + 1 = 1$ — Replace x with 0
when $x = 2$, $y = 2(2) + 1 = 4 + 1 = 5$ and — Replace x with 2
when $x = 3$, $y = 2(3) + 1 = 6 + 1 = 7$ — Replace x with 3

The ordered pairs $(-3,-5)$, $(0,1)$, $(2,5)$, and $(3,7)$ are solutions of the equation $y = 2x + 1$. These ordered pairs are often shown in an x-y table

x	$y = 2x + 1$	Ordered pair
-3	$2(-3) + 1 = -5$	$(-3,-5)$
0	$2(0) + 1 = 1$	$(0,1)$
2	$2(2) + 1 = 5$	$(2,5)$
3	$2(3) + 1 = 7$	$(3,7)$

We now plot these points in figure 7–4.

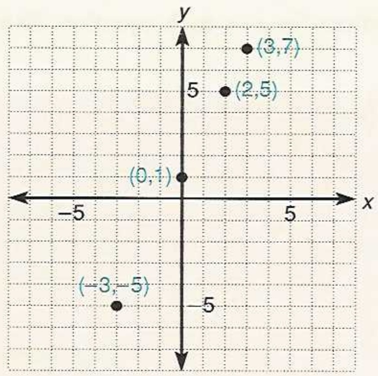


Figure 7–4

> ***Mastery points***
>
> *Can you*
>
> - Determine whether or not an ordered pair is a solution of a given equation?
> - Find the value of one variable, given the value of the other variable?
> - Plot ordered pairs in the rectangular coordinate plane?
> - Plot ordered pair solutions of linear equations?

Exercise 7–1

Determine whether or not the given ordered pairs are solutions of the given equation. See example 7–1 A.

Example Consider $3x + y = 2$; $(1,-1)$, $(2,0)$.

Solution

a.
$$x + y = 2$$
$$3(1) + (-1) = 2 \quad \text{Replace } x \text{ with 1 and } y \text{ with } -1$$
$$3 + (-1) = 2$$
$$2 = 2 \quad \text{(True)}$$
Therefore $(1,-1)$ is a solution.

b.
$$3x + y = 2$$
$$3(2) + (0) = 2 \quad \text{Replace } x \text{ with 2 and } y \text{ with 0}$$
$$6 + 0 = 2$$
$$6 = 2 \quad \text{(False)}$$
Therefore $(2,0)$ is *not* a solution.

1. $y = 3x - 1$; $(1,2)$, $(-1,-4)$, $(2,3)$

2. $y = 2x + 4$; $(-1,3)$, $(0,4)$, $(2,8)$

3. $x + 2y = 3$; $(1,2)$, $(-1,2)$, $(3,0)$

4. $3x - y = 4$; $(1,-1)$, $\left(\frac{1}{3},2\right)$, $(0,-4)$

5. $3y - 4x = 2$; $(1,2)$, $(-2,1)$, $\left(\frac{1}{2},1\right)$

6. $5x - 2y = 6$; $(2,2)$, $(0,-3)$, $(4,-2)$

7. $3x = 2y$; $(2,3)$, $(3,2)$, $(0,0)$

8. $3y = -4x$; $(4,-3)$, $(-4,3)$, $(8,-6)$

9. $x = -4$; $(-4,1)$, $(4,2)$, $(-4,-4)$

10. $y = 3$; $(2,3)$, $(-5,2)$, $\left(\frac{3}{4},3\right)$

11. $x + 5 = 0$; $(3,-5)$, $(-5,3)$, $(-5,8)$

12. $y - 2 = 0$; $(-2,2)$, $\left(\frac{2}{3},-2\right)$, $(5,2)$

Find the value for y corresponding to the given values for x in each equation. Express the answer as an ordered pair. See Example 7–1 B.

Example Consider $2x + y = 1$; $x = 2$, $x = -3$.

Solution a. Let $x = 2$ then

$$\begin{aligned} 2x + y &= 1 \\ 2(2) + y &= 1 && \text{Replace } x \text{ with } 2 \\ 4 + y &= 1 && \text{Solve for } y \\ y &= -3 \end{aligned}$$

The ordered pair is $(2,-3)$.

b. Let $x = -3$ then

$$\begin{aligned} 2x + y &= 1 \\ 2(-3) + y &= 1 && \text{Replace } x \text{ with } -3 \\ -6 + y &= 1 && \text{Solve for } y \\ y &= 7 \end{aligned}$$

The ordered pair is $(-3,7)$.

13. $y = 3x + 2$; $x = 1$, $x = -2$, $x = 0$

14. $y = 4x - 3$; $x = -1$, $x = 2$, $x = 0$

$\boxed{15.}$ $y = 3 - 2x$; $x = 3$, $x = -4$, $x = 0$

16. $y = -5 - x$; $x = \dfrac{1}{5}$, $x = 5$, $x = 0$

17. $3x + y = 4$; $x = 3$, $x = -2$, $x = 0$

18. $y - 4x = 1$; $x = 1$, $x = \dfrac{5}{4}$, $x = 0$

19. $x - 5y = 3$; $x = -2$, $x = 3$, $x = 0$

$\boxed{20.}$ $x + 4y = 0$; $x = -4$, $x = 8$, $x = 0$

21. $5x + 2y = -3$; $x = 1$, $x = -1$, $x = 0$

22. $2x - 3y = 1$; $x = \dfrac{1}{4}$, $x = -4$, $x = 0$

$\boxed{23.}$ $y = 5$; $x = 1$, $x = -6$, $x = 0$

24. $y = -4$; $x = -5$, $x = -4$, $x = 0$

25. $y + 1 = 0$; $x = 7$, $x = -\dfrac{3}{5}$, $x = 0$

26. $y - 4 = 0$; $x = 1$, $x = 2$, $x = 0$

Find the value for x corresponding to the given value for y in each equation. Express the answer as an ordered pair. See example 7–1 B.

Example Consider $2y - 3x = 1$; $y = -4$, $y = 0$.

Solution a. Let $y = -4$, then

$$\begin{aligned} 2y - 3x &= 1 \\ 2(-4) - 3x &= 1 && \text{Replace } y \text{ with } -4 \\ -8 - 3x &= 1 && \text{Solve for } x \\ -3x &= 9 \\ x &= -3 \end{aligned}$$

The ordered pair is $(-3,-4)$.

b. Let $y = 0$, then

$$\begin{aligned} 2y - 3x &= 1 \\ 2(0) - 3x &= 1 && \text{Replace } y \text{ with } 0 \\ 0 - 3x &= 1 && \text{Solve for } x \\ -3x &= 1 \\ x &= -\frac{1}{3} \end{aligned}$$

The ordered pair is $\left(-\dfrac{1}{3},0\right)$

27. $x = 2y + 3$; $y = 1$, $y = -2$, $y = 0$

$\boxed{28.}$ $x = -3y + 1$; $y = -1$, $y = 2$, $y = 0$

29. $3y - 2x = 0$; $y = 2$, $y = -4$, $y = 0$

30. $2x + y = 3$; $y = -3$, $y = 5$, $y = 0$

$\boxed{31.}$ $x = 1$; $y = -2$, $y = 7$, $y = 0$

32. $x + 7 = 0$; $y = -1$, $y = 3$, $y = 0$

Solve the following word problems.

33. The total cost c in dollars of producing x units of a certain commodity is given by the equation $c = 2x + 20$. Find the cost of producing (a) 75; (b) 300; (c) 1,000 units of the commodity. Write the answers as ordered pairs (x,c).

34. In exercise 33, find the number of units produced when the total cost is (a) \$430; (b) \$700; (c) \$1,400. Write the answers as ordered pairs (x,c).

35. The cholesterol level in the blood, y, is related to the dosage of a new anticholesterol drug in grams, x, by the equation $y = 240 - 2x$. Find the cholesterol level in the blood when the dosage is (a) 2 grams, (b) 12 grams, (c) 0 grams. Write the answers as ordered pairs (x,y).

36. In exercise 35, determine the number of grams in the dosage when the cholesterol level in the blood is (a) 200, (b) 0, (c) 210. Write the answers as ordered pairs (x,y).

37. Suppose the equation $y = 3x + 20$ represents the number of students present in a mathematics class, where x represents the number of hours of study required and y represents the number of students present. Find the number of students present when the class requires (a) 0 hours of study, (b) 5 hours of study, (c) 3 hours of study. Write the answers as ordered pairs (x,y).

38. In exercise 37, find the number of hours of study required when there are (a) 26 students present, (b) 23 students present, (c) 32 students present. Write the answers as ordered pairs (x,y).

39. If the distance, y, (in miles) that Mary Jane travels in x hours of driving is given by the equation $y = 55x$, how far does Mary Jane travel in (a) 3 hours, (b) 8 hours, (c) 5 hours and 12 minutes $\left(5\frac{1}{5} \text{ hours}\right)$? Write the answers as ordered pairs (x,y).

40. In exercise 39, how many hours does it take Mary Jane to drive (a) 110 miles, (b) 385 miles, (c) 500 miles? Write the answers as ordered pairs (x,y).

Plot the following ordered pairs on a rectangular coordinate plane. See example 7–1 C.

41. $(2,4)$ **42.** $(5,1)$ **43.** $(-1,3)$ **44.** $(-4,4)$ **45.** $(-6,-1)$

46. $(-2,-3)$ **47.** $(0,4)$ **48.** $(0,2)$ **49.** $(5,0)$ **50.** $(-4,0)$

51. $(-7,0)$ **52.** $(0,0)$ **53.** $\left(\frac{1}{2},3\right)$ **54.** $\left(\frac{2}{3},-2\right)$ **55.** $\left(\frac{3}{2},0\right)$

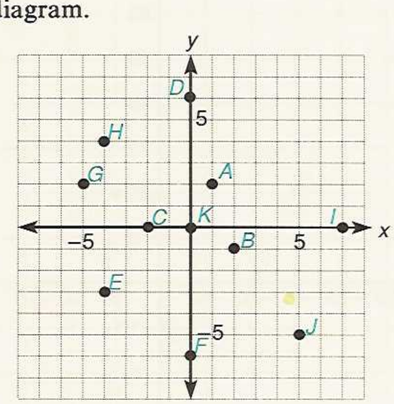
Determine the coordinates, (x,y), of the given points in the diagram.

56. A **57.** B **58.** C

59. D **60.** E **61.** F

62. G **63.** H **64.** I

65. J **66.** K



State the quadrant in which each point lies.

Example $(-1,3)$ lies in the second quadrant because the x-component is negative and the y-component is positive.

67. $(-2,-5)$ **68.** $(4,-1)$ **69.** $(5,3)$ **70.** $(-7,-9)$ **71.** $\left(\frac{1}{2},-4\right)$

72. $\left(-\frac{2}{3},7\right)$ **73.** $\left(-\frac{5}{2},-\frac{3}{4}\right)$ **74.** $\left(\frac{7}{8},-\frac{7}{8}\right)$ **75.** $(5,-14)$ **76.** $(-3,-3)$

Solve the following word problems.

77. In what quadrant does a point (x,y) lie if (a) $x > 0$ and $y < 0$, (b) $x < 0$ and $y > 0$, (c) $x < 0$ and $y < 0$, (d) $x > 0$ and $y > 0$?

78. What is the value of x for any point on the y-axis?

79. What is the value of y for any point on the x-axis?

80. Using the diagram for exercises 56–66, what is the *abscissa* of each of the points A, C, E, G, I, and K?

81. Using the diagram for exercises 56–66, what is the *ordinate* of each of the points B, D, F, H, and J?

$\boxed{82.}$ Plot five points whose abscissa is -3. Connect the plotted points. Describe the resulting figure.

83. Using the same axes as in exercise 82, plot five points whose ordinate is 4. Connect the plotted points. Describe the resulting figure.

84. Choose five ordered pairs whose first and second coordinates are the same. Plot these points and connect the points. What kind of figure do we get? In what quadrants does the figure lie?

85. Choose five ordered pairs whose first component is the opposite of the second component. Plot the points and connect them. What kind of figure do we get? In what quadrants does the figure lie?

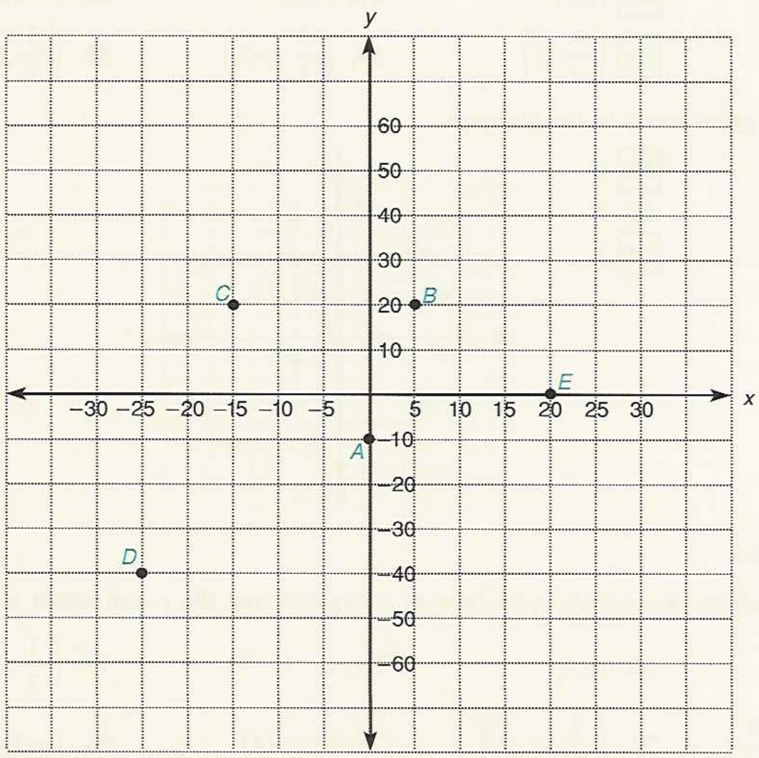
Plot the points in the following exercises using the indicated scale.

Note Exercises 86 to 90 require the use of different scales on the two axes. This is sometimes necessary in certain problems, especially in applications in the sciences and other fields.

Example 1 square = 5 units (on x-axis)
1 square = 10 units (on y-axis)

Note A "square" is one unit on a grid.

$A(0,-10)$, $B(5,20)$, $C(-15,20)$, $D(-25,-40)$, $E(20,0)$



86. 1 square = 10 units
$A(20,20)$, $B(-30,40)$, $C(0,-50)$, $D(40,0)$, $E(50,-70)$

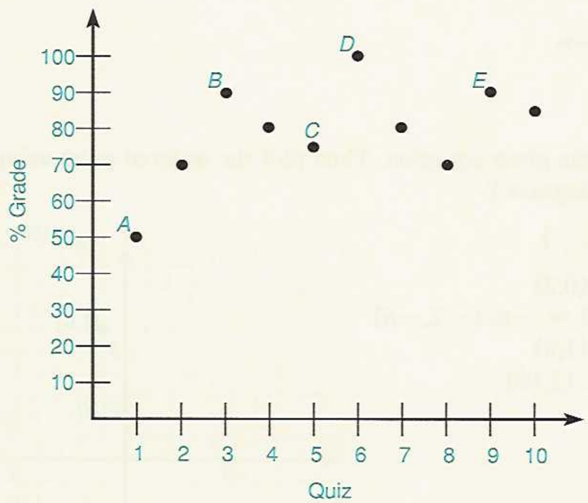
87. 1 square = 1 unit (on x-axis)
1 square = 5 units (on y-axis)
$A(4,10)$, $B(-6,35)$, $C(5,5)$, $D(0,-45)$, $E(-3,-50)$

88. 1 square = 5 units (on x-axis)
1 square = 1 unit (on y-axis)
$A(-20,3)$, $B(-5,5)$, $C(40,-6)$, $D(25,0)$, $E(-20,-4)$

89. 1 square = 10 units
$A(-35,20)$, $B(35,-40)$, $C(25,100)$, $D(0,0)$, $E(-40,90)$

90. 1 square = 25 units (on x-axis)
1 square = 100 units (on y-axis)
$A(75,-600)$, $B(200,-200)$, $C(0,-100)$, $D(-175,500)$, $E(-225,0)$

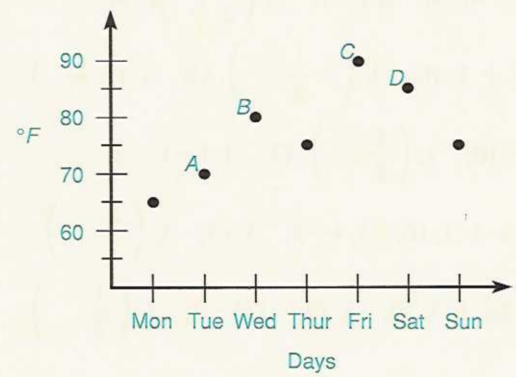
Example The following set of points represents the percent grades Bruce received on ten algebra quizzes.



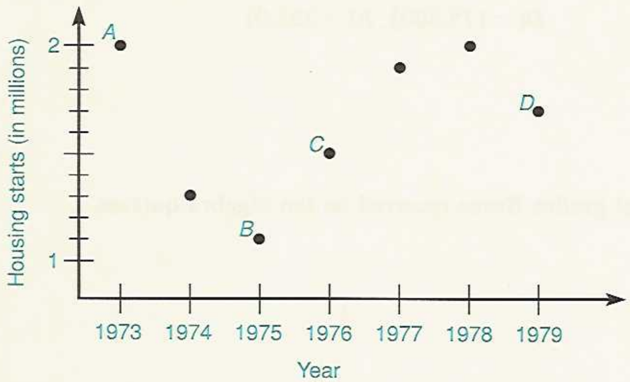
Find the coordinates of points A, B, C, D, and E and interpret the coordinates.

Solution $A(1,50)$: Grade was 50 percent on first quiz
$B(3,90)$: Grade was 90 percent on third quiz
$C(5,75)$: Grade was 75 percent on fifth quiz
$D(6,100)$: Grade was 100 percent on sixth quiz
$E(9,90)$: Grade was 90 percent on ninth quiz

91. The following sets of points represent the average temperature in Detroit during a seven-day period last summer. Find the approximate coordinates of points A, B, C, and D and interpret the coordinates.



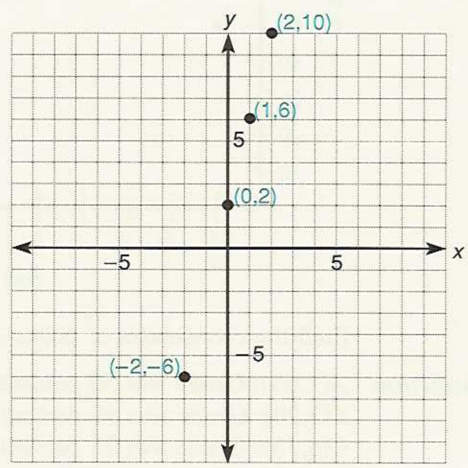
92. The following sets of points represent the approximate number of housing starts in the United States for the years 1973 to 1979. Find the coordinates of points A, B, C, and D and interpret the coordinates.



Find the missing component in each ordered pair using the given equation. Then plot the ordered pairs using a separate coordinate system for each problem. (See the diagram.)

Example $y = 4x + 2$; $(0,\quad), (-2,\quad), (1,\quad), (2,\quad)$

Solution When $x = 0, y = 4(0) + 2 = 0 + 2 = 2; (0,2)$
$x = -2, y = 4(-2) + 2 = -8 + 2 = -6; (-2,-6)$
$x = 1, y = 4(1) + 2 = 4 + 2 = 6; (1,6)$
$x = 2, y = 4(2) + 2 = 8 + 2 = 10; (2,10)$



93. $y = x + 4; (0,\quad), (-4,\quad), \left(\frac{5}{2},\quad\right), (2,\quad)$

94. $y = x - 3; (0,\quad), (3,\quad), (1,\quad), \left(-\frac{1}{2},\quad\right)$

95. $y = 2x + 1; (0,\quad), \left(-\frac{1}{2},\quad\right), (2,\quad), (-1,\quad)$

96. $y = 3x - 4; (0,\quad), (-1,\quad), \left(-\frac{1}{3},\quad\right), (2,\quad)$

97. $y = 2x; (0,\quad), \left(\frac{1}{2},\quad\right), (2,\quad), (-1,\quad)$

98. $y = 3x; (0,\quad), (3,\quad), (-3,\quad), \left(\frac{1}{3},\quad\right)$

99. $y = -x + 3; (0,\quad), (-3,\quad), (3,\quad), \left(\frac{5}{2},\quad\right)$

100. $y = -x + 1; (0,\quad), (1,\quad), (-2,\quad), (-4,\quad)$

101. $y = -2x + 3; (0,\quad), (3,\quad), (-2,\quad), \left(\frac{3}{2},\quad\right)$

102. $3x + y = 1; (0,\quad), \left(\frac{1}{3},\quad\right), (-2,\quad), (1,\quad)$

103. $y = 4 - x; (0,\quad), (-3,\quad), (2,\quad), (3,\quad)$

104. $2y - 3x = 2; (0,\quad), \left(\frac{2}{3},\quad\right), (-2,\quad), (2,\quad)$

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Review exercises

Perform the indicated operations. See sections 6–1 and 6–2.

1. $\dfrac{x^2 - 4}{3x} \cdot \dfrac{6x^2}{x - 2}$
2. $\dfrac{2x - 1}{x + 3} \div \dfrac{4x^2 - 1}{x^2 + 6x + 9}$
3. $\dfrac{4}{x - 2} - \dfrac{3}{x + 1}$
4. Find $-\,|-8|$. See section 1–3.
5. Multiply $x^2 \cdot x \cdot x^0$. See section 3–3.
6. Multiply $(-2)(-3)(4)(0)$. See section 1–6.

7–2 ■ Graphs of linear equations

Graph of a linear equation

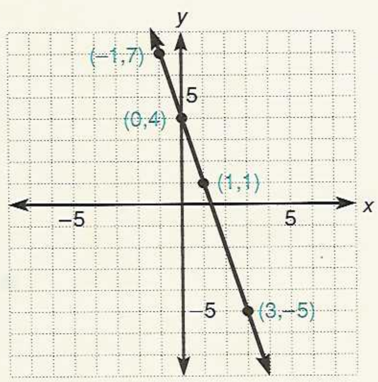


Figure 7–5

In section 7–1, we learned that there are infinitely many ordered pairs that will satisfy an equation in two variables. That is, given the linear equation $3x + y = 4$, we can find many ordered pairs (as many as we wish) that are solutions of the equation. To list all of these solutions is impossible. However, these solutions can be represented geometrically by a graph of the ordered pairs that are solutions.

In section 7–1, we plotted the graphs of several of the ordered pairs that satisfied the given equations. To illustrate again, consider the equation $3x + y = 4$. In figure 7–5, we plot the ordered pairs

$$(0,4),\ (1,1),\ (-1,7),\ \text{and } (3,-5)$$

that are solutions of the equation.

Connecting the points, we find they all lie on the same straight line. We have drawn arrowheads in each direction at each end of the line to indicate that the line goes on indefinitely in each direction. *Any point whose coordinates satisfy the equation* $3x + y = 4$ *will lie on this line,* and *the coordinates of any point on this line will satisfy the equation.* We have a graphical representation of a portion of the solutions of the equation. The straight line in figure 7–5 is called the *graph of the equation* $3x + y = 4$.

Note Any point that *does not* lie on this line has coordinates that will not satisfy the equation $3x + y = 4$ and so *does not* represent a solution of the equation. For example, the point $(-1,1)$ does not lie on the line. If we substitute, we obtain

$$\begin{aligned} 3(-1) + 1 &= 4 \\ -3 + 1 &= 4 \\ -2 &= 4 \quad \text{(False)} \end{aligned}$$

Straight line

In general, the graph of *any* linear equation in two variables is a *straight line.* A geometric fact we now use is that *through any two given points in the plane we can draw one and only one straight line.* Thus, since we know the graph of a linear equation in two variables is a straight line, we can determine the graph of the equation using only two points. However, it is a good idea to find a third point as a check on our work. (Remember, the word *line* appears in the name *line*ar equation.)

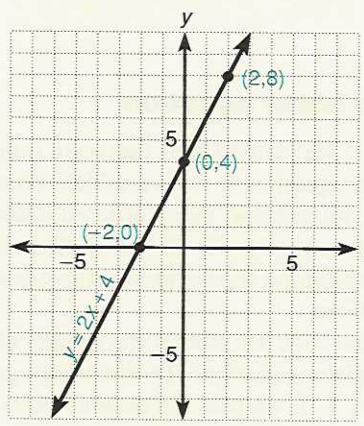
■ Example 7–2 A

Graph the equation $y = 2x + 4$.

We choose *three* arbitrary values of x and find corresponding values for y by substituting the value of x and solving for y.

$$\begin{aligned} \text{Let } x = 0, &\text{ then } y = 2(0) + 4 = 0 + 4 = 4 \\ x = -2, &\text{ then } y = 2(-2) + 4 = -4 + 4 = 0 \\ x = 2, &\text{ then } y = 2(2) + 4 = 4 + 4 = 8 \end{aligned}$$

Now we graph the points (0,4), (−2,0), and (2,8) and draw a straight line through the points.



The x- and y-intercepts

Notice the graph of $y = 2x + 4$ crosses the y-axis at (0,4) and the x-axis at (−2,0). The points (0,4) and (−2,0) are called the *y-intercept* and the *x-intercept,* respectively. Since we need only two points to sketch the graph of a linear equation in two variables, in many cases we use the x- and y-intercepts. Observe from the example that when the line crosses the x-axis, the value of y is zero. When the line crosses the y-axis, the value of x is zero.

Finding x- and y-intercepts

1. To find the x-intercept, we let $y = 0$ and find the corresponding value of x. This is the point $(x,0)$.
2. To find the y-intercept, we let $x = 0$ and find the corresponding value for y. This is the point $(0,y)$.

■ Example 7–2 B

Graph the following linear equations using x- and y-intercepts.

1. $y = -2x + 3$

Let $x = 0$ to find the y-intercept; let $y = 0$ to find the x-intercept.

$y = -2x + 3$		$y = -2x + 3$	
$y = -2(0) + 3$	Replace x with 0	$(0) = -2x + 3$	Replace y with 0
$y = 0 + 3$	Multiply as indicated	$2x = 3$	Add $2x$ to each member
$y = 3$		$x = \dfrac{3}{2}$	

The point (0,3) is the y-intercept.

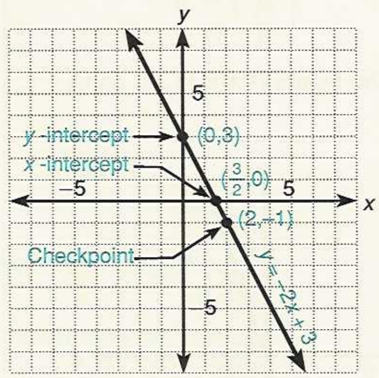
The point $\left(\dfrac{3}{2},0\right)$ is the x-intercept.

To find another point, choose $x = 2$.

$y = -2x + 3$
$y = -2(2) + 3$ Replace x with 2
$y = -4 + 3$ Multiply as indicated
$y = -1$

The third point is $(2,-1)$.

We now plot the three points $(0,3)$, $\left(\frac{3}{2},0\right)$ and $(2,-1)$ and draw a straight line through them.



Note The components of these points are often stated in a table of related values as illustrated in the following examples.

2. $3y - 2x = 9$

Let $x = 0$ to find the y-intercept; let $y = 0$ to find the x-intercept.

When $x = 0$

$3y - 2x = 9$
$3y - 2(0) = 9$ Replace x with 0
$3y - 0 = 9$
$3y = 9$
$y = 3$

The point $(0,3)$ is the y-intercept.

When $y = 0$

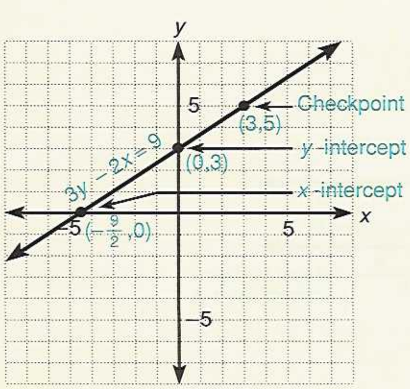
$3y - 2x = 9$
$3(0) - 2x = 9$ Replace y with 0
$0 - 2x = 9$
$-2x = 9$
$x = -\frac{9}{2}$

The point $\left(-\frac{9}{2},0\right)$ is the x-intercept.

For the third point, let $x = 3$.

$3y - 2x = 9$
$3y - 2(3) = 9$ Replace x with 3
$3y - 6 = 9$
$3y = 15$
$y = 5$

The checkpoint is $(3,5)$.



Plot the points $(0,3)$, $\left(-\frac{9}{2},0\right)$, and $(3,5)$ and draw a straight line through them.

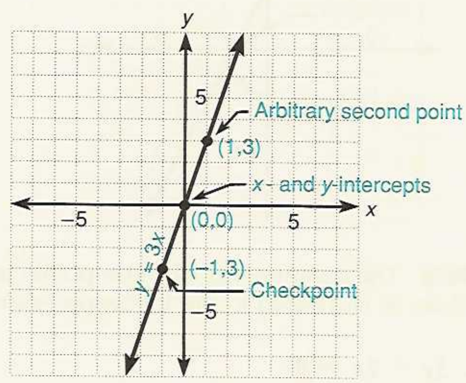
In the previous examples, the x- and y-intercepts were different points. For some equations, the x- and y-intercepts are the same point, as demonstrated in example 3.

3. $y = 3x$

If we let $x = 0$, then $y = 3(0) = 0$, giving the ordered pair (0,0). When $y = 0$, then $0 = 3x$ and $x = 0$, giving the same point (0,0). We must choose two additional values for x or y. Let $x = 1$ and $x = -1$.

x	**y = 3x**	**Ordered pair (x,y)**	
0	$3(0) = 0$	(0,0)	x- and y-intercepts
1	$3(1) = 3$	(1,3)	Arbitrary second point
−1	$3(-1) = -3$	(−1,−3)	Checkpoint

Plot the points (0,0), (1,3), and (−1,−3) and draw a straight line through them.



To generalize, any linear equation that can be written in the form

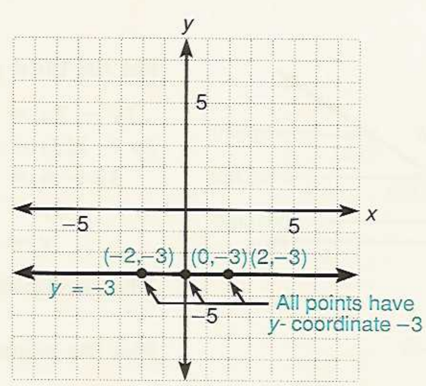
$$y = kx \quad \text{or} \quad x = ky$$

where k is a real number, will pass through the origin (0,0). The next two examples show what happens when one of the variables is missing.

4. $y = -3$

Recall that this equation could be written $y = 0 \cdot x - 3$ and that for *any* value of x we might choose, y is *always* equal to -3. Therefore, we choose any three values for x and obtain $y = -3$.

x	y
−2	−3
0	−3
2	−3

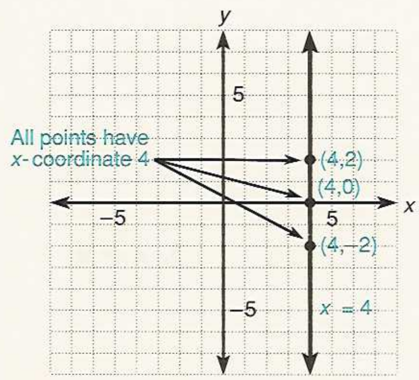


Note The graph has a y-intercept, -3, but no x-intercept. The graph is a horizontal straight line passing through the point $(0,-3)$ (parallel to the x-axis). In fact, the graph of any equation of the form $y = b$ will be a horizontal line passing through the point with coordinates $(0,b)$, the y-intercept.

5. $x = 4$

Recall that this equation can be written $x = 0 \cdot y + 4$ and that the value of x will be 4 for *any* value of y.

x	y
4	-2
4	0
4	2



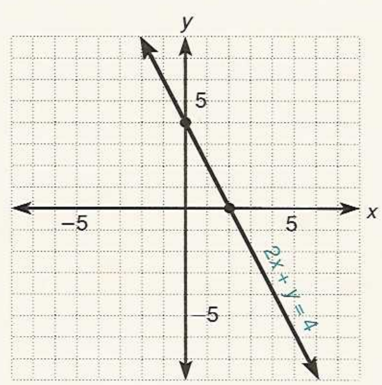
Note The graph has an x-intercept, 4, but no y-intercept. The graph is a *vertical* straight line (parallel to the y-axis) passing through the point with coordinates $(4,0)$. In fact, the graph of any equation of the form $x = a$ will be a vertical line passing through the point with coordinates $(a,0)$, the x-intercept.

► ***Quick check*** a. Find the x- and y-intercepts for the graph of $5x + 3y = 15$
b. Graph the linear equation $2y - 3x = 12$ using the x- and y-intercepts. ■

We now summarize the different forms that linear equations might take.

1. $ax + by = c$ Graph by finding the x-intercept (let $y = 0$), the y-intercept (let $x = 0$), and a third checkpoint by choosing any value for x or y not yet used.

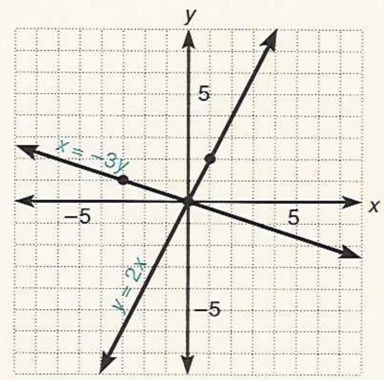
Example:
$2x + y = 4$



2. $y = kx$ or $x = ky$

Graph goes through the origin (0,0). Find two other points by choosing values for x or y other than 0.

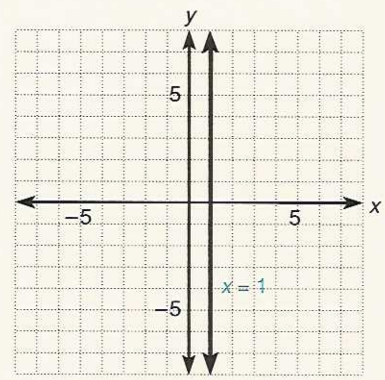
Examples:
$y = 2x$ and
$x = -3y$



3. $x = a$

Graph is a vertical line through $(a,0)$.

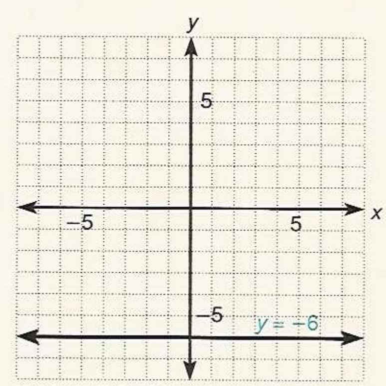
Example: $x = 1$



4. $y = b$

Graph is a horizontal line through $(0,b)$.

Example: $y = -6$



Mastery points

Can you

- Plot the graph of linear equations using ordered pairs?
- Find the x- and y-intercepts of a linear equation?
- Plot the graph of linear equations using the x- and y-intercepts?
- Plot graphs of the equations $x = a$ and $y = b$, where a and b are constants?

Exercise 7–2

Find the x- and y-intercepts. Write ordered pairs representing the points where the line crosses the axes. See example 7–2 B.

Example $5x + 3y = 15$

Solution Let $x = 0$, then $5(0) + 3y = 15$ Replace x with 0

$$0 + 3y = 15$$
$$3y = 15$$
$$y = 5 \quad \text{y-intercept}$$

Let $y = 0$, then $5x + 3(0) = 15$ Replace y with 0

$$5x + 0 = 15$$
$$5x = 15$$
$$x = 3 \quad \text{x-intercept}$$

The line crosses the y-axis at (0,5) and the x-axis at (3,0).

1. $y = 2x + 4$
2. $y = 5x - 10$
3. $y = 3x + 1$
4. $y = 2x - 3$
5. $2x + 3y = 6$
6. $x + 4y = 12$
7. $2x + 5y - 11 = 0$
8. $x - y = 4$
9. $y = 5x$
10. $y = -2x$
11. $3x - 2y = 0$
12. $y - 4x = 0$
13. $(1.2)x + (2.4)y = 4.8$
14. $(0.3)x - (0.4)y = 0.7$
15. $y = \dfrac{1}{2}x - \dfrac{3}{2}$
16. $y = \dfrac{2}{3}x - \dfrac{1}{3}$

Plot the graphs of the given linear equations using the x- and y-intercepts. See example 7–2 B.

Example $2y - 3x = 12$

Solution

When $x = 0$

$$2y - 3x = 12$$
$$2y - 3(0) = 12 \quad \text{Replace } x \text{ with } 0$$
$$2y - 0 = 12$$
$$2y = 12$$
$$y = 6 \quad \text{y-intercept}$$

The point (0,6) is the y-intercept.

When $y = 0$

$$2y - 3x = 12$$
$$2(0) - 3x = 12 \quad \text{Replace } y \text{ with } 0$$
$$0 - 3x = 12$$
$$-3x = 12$$
$$x = -4 \quad \text{x-intercept}$$

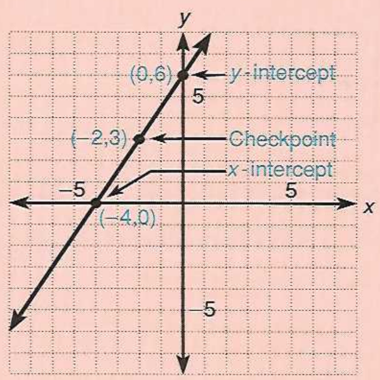
The point $(-4,0)$ is the x-intercept.

When $x = -2$

$$2y - 3x = 12$$
$$2y - 3(-2) = 12 \quad \text{Replace } x \text{ with } -2$$
$$2y - (-6) = 12$$
$$2y + 6 = 12$$
$$2y = 6$$
$$y = 3$$

The checkpoint is $(-2,3)$.

We now plot the points (0,6), $(-4,0)$, and $(-2,3)$ and draw a line through them.



17. $y = 3x + 6$ 18. $y = x + 5$ 19. $y = x - 2$ 20. $y = 2x + 4$

21. $y = 2x - 8$ 22. $y = 3x$ 23. $y = x$ 24. $y = -3x$

25. $y = -2x$ 26. $x + y = 0$ 27. $y - 2x = 0$ 28. $x - 3y = 0$

29. $4x - 3y = 12$ 30. $3x - 2y = 6$ 31. $2y + 5x = 10$ 32. $2x + 2y = 3$

33. $5x - 6y = 30$ 34. $3y - 5x = 15$ 35. $y = 6$ 36. $y = -2$

37. $x = 5$ 38. $x = -2$ 39. $x = 0$ 40. $y = 0$

Solve the following equations for y in terms of x. Write the equations in the form $y = mx + b$, where m and b are rational numbers. Identify m and b.

Example $3y - 2x = 4$

Solution

$$3y = 2x + 4 \quad \text{Add } 2x \text{ to each member}$$

$$y = \frac{2}{3}x + \frac{4}{3} \quad \text{Divide each member by 3}$$

$$m = \frac{2}{3} \text{ and } b = \frac{4}{3}$$

41. $y - 2x + 7 = 0$ 42. $y - 3x - 4 = 0$ 43. $3y - 4x = 9$ 44. $5y - 2x = 7$

45. $7x + 3y = 10$ 46. $5x + 3y = 4$ 47. $x - 5y + 7 = 0$ 48. $x - 3y + 9 = 0$

49. $5y - 8x + 14 = 0$ 50. $7x + 5y - 11 = 0$

51. Plot the graph of the equation $y = -2x + b$ for (a) $b = 5$, (b) $b = 0$, and (c) $b = -3$ all on the same coordinate system.

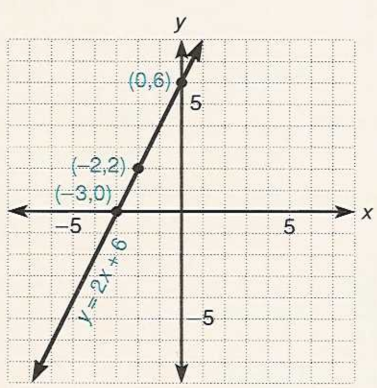
52. Plot the graph of the equation $y = mx + 1$ for (a) $m = 1$, (b) $m = \frac{1}{2}$, and (c) $m = -2$ all on the same coordinate system.

Write an equation for each of the statements in exercises 53 to 56 and plot the graph of the equation.

Example The value of y is 6 more than twice the value of x.

Solution The equation is $y = 2x + 6$.

x	y	
0	6	y-intercept
−3	0	x-intercept
−2	2	Checkpoint



53. The value of y is 3 less than two times the value of x.

54. The value of x is 4 more than the value of y.

55. Two times x taken away from three times y is 6.

56. Five times x less the product of 2 and y gives 20.

Review exercises

Reduce the following expressions to lowest terms. See sections 1–1 and 5–2.

1. $\frac{18}{15}$

2. $\frac{x - 3}{x^2 + x - 12}$

3. $\frac{x^2 - 8x + 16}{x^2 + x - 20}$

4. Given $\frac{a - b}{c - d}$, evaluate the expression when $a = 2$, $b = 1$, $c = -3$, and $d = 2$. See section 5–1.

5. Given $y = mx + b$, find y when $m = -3$ and $b = 6$. See section 2–2.

6. If the product of a number and three times that number is 48, find the number. See section 4–8.

7–3 ■ The slope of a line

The slope

Consider the portions of the two roadways denoted by R_1 and R_2 (read "R sub-one" and "R sub-two") shown in figure 7–6.

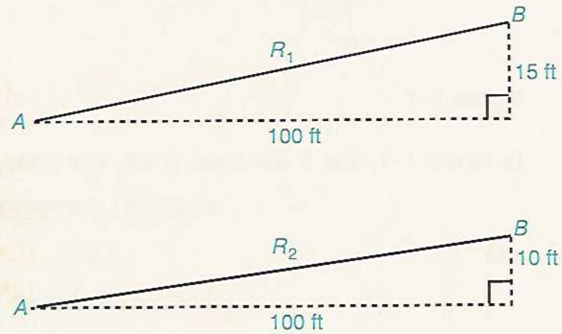


Figure 7–6

We would say roadway R_1 is "steeper" than roadway R_2. In moving from point A to point B on each roadway, a horizontal change in position of 100 feet, the vertical change in position is

15 feet on roadway R_1

and

10 feet on roadway R_2

If we measure this "steepness" by the ratio

$$\frac{\text{vertical change}}{\text{horizontal change}}$$

the roadway

$$R_1 \text{ has "steepness"} = \frac{15 \text{ ft}}{100 \text{ ft}} = \frac{3}{20}$$

and

$$R_2 \text{ has "steepness"} = \frac{10 \text{ ft}}{100 \text{ ft}} = \frac{1}{10}$$

Note $\frac{3}{20}$ is greater than $\frac{1}{10}$, so R_1 is "steeper" than R_2.

When applying this concept to any straight line, "steepness" is called the **slope** of the line. Thus, the slope of any line L is given by

$$\text{slope} = \frac{\text{vertical change}}{\text{horizontal change}}$$

Observe that the *slope is a ratio.* (See figure 7–7.)

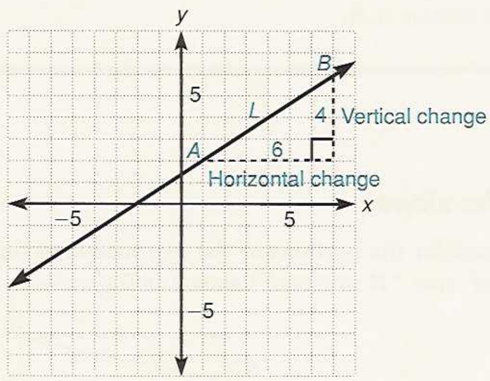


Figure 7–7

In figure 7–7, line L has from point A to point B

$$\text{a vertical change} = 4 \text{ units}$$

and

$$\text{a horizontal change} = 6 \text{ units}$$

Thus, the

$$\text{slope of } L = \frac{\text{vertical change}}{\text{horizontal change}} = \frac{4}{6} = \frac{2}{3}$$

To obtain the slope of a nonvertical straight line, given points P_1 and P_2 (read "P sub-one" and "P sub-two") which have coordinates (x_1,y_1) and (x_2,y_2), respectively, use the following definition.

Definition of the slope of a nonvertical line

The slope m of the nonvertical line through the points $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (x_1 \neq x_2)$$

Concept

The slope of a nonvertical line is determined by dividing the change in y-values by the change in x-values of any two points on the line.

See figure 7–8.

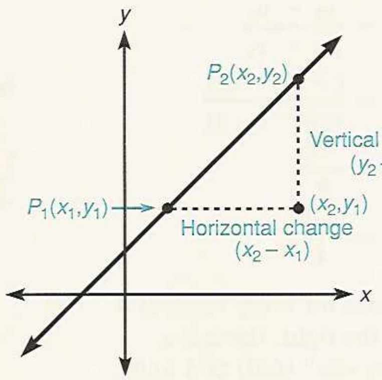


Figure 7–8

Note The vertical change is sometimes called the *rise* and the horizontal change is called the *run.* Thus, slope m can be defined

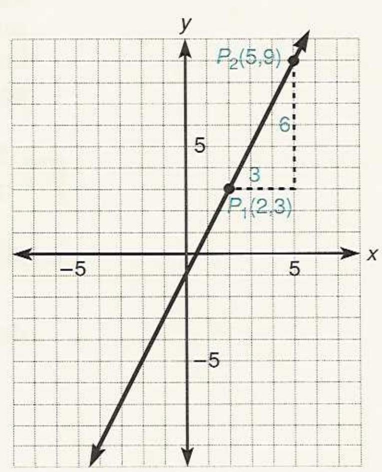
$$m = \frac{\text{rise}}{\text{run}}$$

■ *Example 7–3 A*

1. Find the slope of the line through the points $P_1(2,3)$ and $P_2(5,9)$.

$$\begin{aligned}\text{slope } m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{(9) - (3)}{(5) - (2)} \quad \text{Replace } y_2 \text{ with 9, } y_1 \text{ with 3, } x_2 \text{ with 5, and } x_1 \text{ with 2} \\ &= \frac{6}{3} \\ &= 2\end{aligned}$$

This means that for every unit moved to the right, there is a rise of 2 units.



Note The x- and y-values may be subtracted in any order as long as the coordinates of one point are in the same position in the numerator and the denominator. Thus,

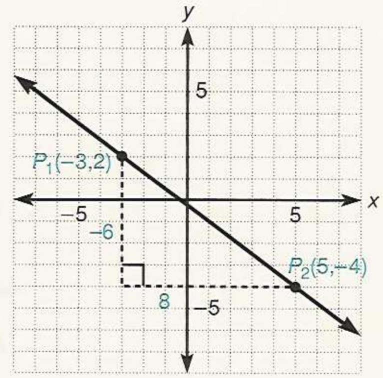
$$m = \frac{3 - 9}{2 - 5} = \frac{-6}{-3} = 2$$

in which we used

$$m = \frac{y_1 - y_2}{x_1 - x_2}$$

2. Find the slope of the line through points $P_1(-3,2)$ and $P_2(5,-4)$.

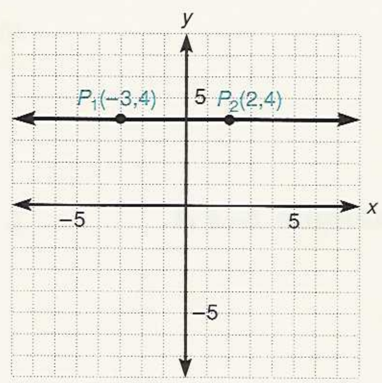
$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-4) - (2)}{(5) - (-3)} = \frac{-6}{8} = \frac{-3}{4}$$



This means for every move of 4 units to the right, there is a "negative rise" (fall) of 3 units.

3. Find the slope of the horizontal line through $P_1(-3,4)$ and $P_2(2,4)$.

$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(4) - (4)}{(2) - (-3)} = \frac{0}{5} = 0$$



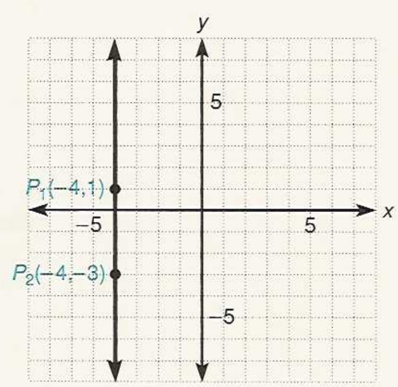
The slope $m = 0$ and the line is a horizontal line. Recall that the equation of any horizontal line is of the form $y = b$, or $y - b = 0$, where b is the y-intercept. In this case, the equation of the horizontal line is $y = 4$. Therefore we can generalize as follows:

Slope of a horizontal line

The slope m of any horizontal line having equation $y = b$ is $m = 0$.

4. Find the slope of the vertical line through $P_1(-4,1)$ and $P_2(-4,-3)$.

$$\text{slope } m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-3) - (1)}{(-4) - (-4)} = \frac{-4}{0} \quad \text{(Undefined)}$$



The line is a vertical line and the slope is undefined. Recall that the equation of any vertical line is of the form $x = a$, where a is the x-intercept. In this case, the equation of the vertical line is $x = -4$.

Slope of a vertical line

Any vertical line having equation $x = a$ *has undefined slope.*

▶ ***Quick check*** Find the slope of the line passing through the points $(-2,1)$ and $(3,4)$. Draw the graph of the line. ■

It is important for us to realize that the *slope of a nonvertical line is the same no matter what two points on the line we use to compute the slope.* Consider figure 7–9.

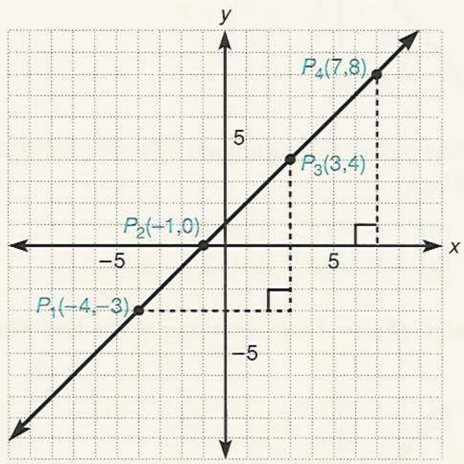


Figure 7–9

$$\text{Using } P_1 \text{ and } P_3,\ m = \frac{(4) - (-3)}{(3) - (-4)} = \frac{7}{7} = 1 \text{ and}$$
$$\text{using } P_2 \text{ and } P_4,\ m = \frac{(8) - (0)}{(7) - (-1)} = \frac{8}{8} = 1$$

We obtain the same slope, $m = 1$

The slope of a line through two plotted points

Suppose we are given the graph of a line and we wish to determine its slope. (See figure 7–10.) We inspect the graph and look for two points on the line that apparently meet the grid lines at points of intersection. In figure 7–10, we can see that the graph goes through the points $(-1,4)$ and $(3,-2)$. Then the slope m is

$$m = \frac{(-2) - (4)}{(3) - (-1)} = \frac{-6}{4} = \frac{-3}{2}$$

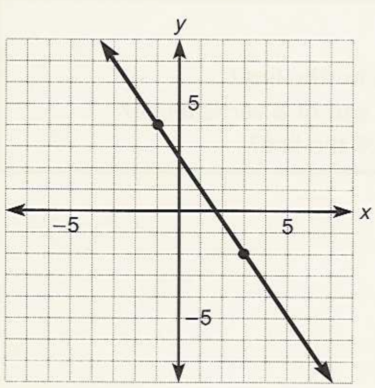
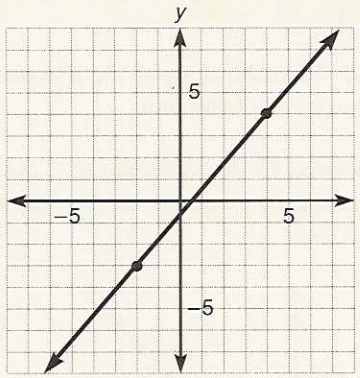


Figure 7–10

■ Example 7–3 B

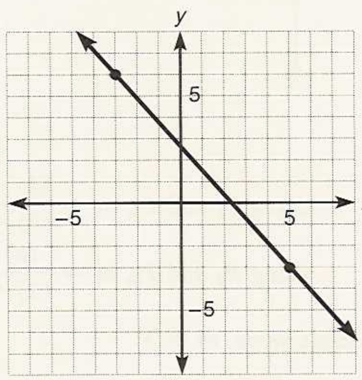
Find the slope of the line whose graph is plotted.



The line passes through points whose coordinates are (4,4) and $(-2,-3)$. Therefore

$$m = \frac{(4) - (-3)}{(4) - (-2)} = \frac{4 + 3}{4 + 2} = \frac{7}{6}$$

▶ ***Quick check*** Find the slope of the line whose graph is plotted.



■

Mastery points

Can you

- Find the slope of a line given two points on the line?
- Remember the slope of a horizontal line and of a vertical line?
- Find the slope of a line given the graph of the line?

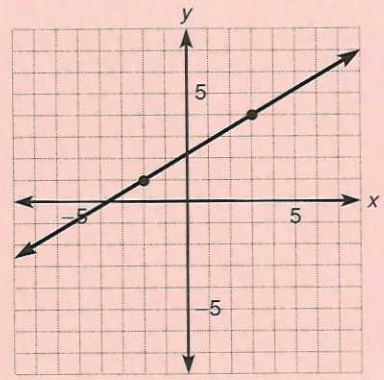
Exercise 7–3

Find the slope of the line passing through each of the following pairs of points. Draw the graph of the line. See example 7–3 A.

Example $(-2,1)$ and $(3,4)$

Solution $m = \dfrac{(4) - (1)}{(3) - (-2)}$

$= \dfrac{3}{5}$



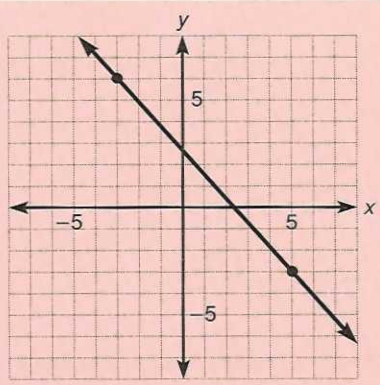
1. $(5,2), (3,3)$
2. $(4,3), (2,2)$
3. $(-4,-1), (2,3)$
4. $(-2,3), (5,5)$
5. $(-4,3), (2,3)$
6. $(4,-4), (2,-4)$
7. $(6,-7), (6,1)$
8. $(-1,4), (-1,-3)$

Find the slope of the line passing through each of the following pairs of points. See example 7–3 A.

9. $(-3,1), (0,3)$
10. $(4,0), (-2,8)$
11. $(-3,9), (-3,-5)$
12. $(5,0), (-3,0)$
13. $(5,6), (3,4)$
14. $(0,0), (4,3)$
15. $(0,7), (0,-8)$
16. $(4,-8), (0,-2)$
17. $(-8,-3), (-1,-2)$
18. $(-6,-5), (-4,-3)$
19. $(-10,4), (2,-5)$
20. $(7,-3), (8,-7)$
21. $(5,7), (-6,-3)$
22. $(9,4), (-1,-1)$

Find the slope of each of the following lines. See example 7–3 B.

Example



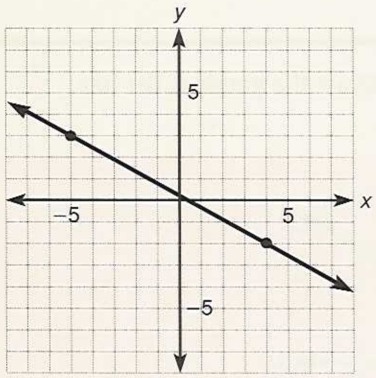
Solution We note that the line passes through the points $(-3,6)$ and $(5,-3)$.

Then slope $m = \dfrac{(6) - (-3)}{(-3) - (5)}$

$= \dfrac{9}{-8}$

$= -\dfrac{9}{8}$

23.



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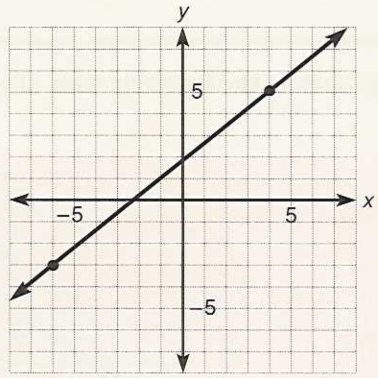

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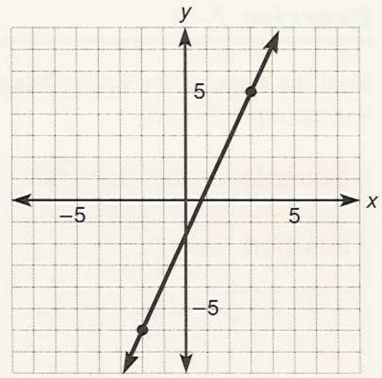

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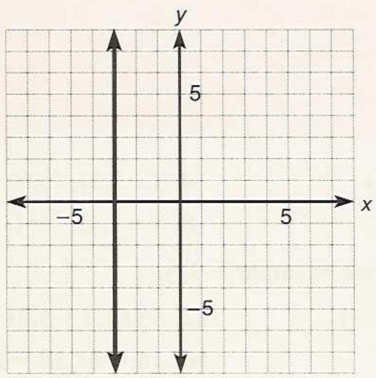
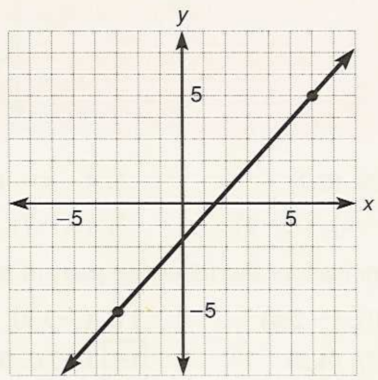


27.



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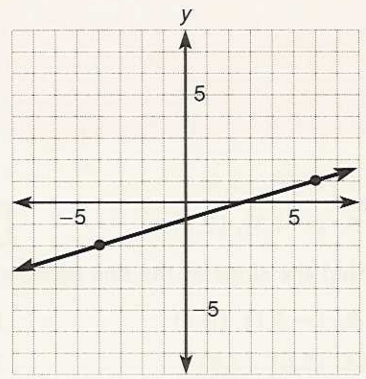
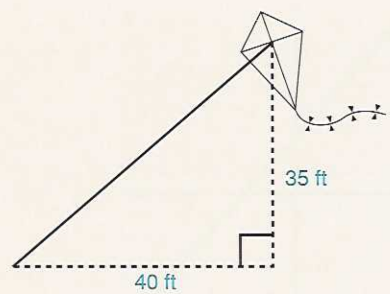


Example A boy is flying his kite. The kite is 35 feet above the ground, and the distance from the boy to a point directly below the kite is 40 feet. If the string from the boy to the kite is a straight line, what is the slope of the string?

Solution Using $m = \dfrac{\text{vertical change}}{\text{horizontal change}}$, the vertical change is 35 feet and the horizontal change is 40 feet, then

$$m = \frac{35 \text{ ft}}{40 \text{ ft}} = \frac{7}{8}$$



29. The roof of a home rises vertically a distance of 8 feet through a horizontal distance of 12 feet. Find the pitch (slope) of the roof.

30. The roof of a factory rises vertically 6 feet through a horizontal run of 27 feet. What is the pitch of the roof?

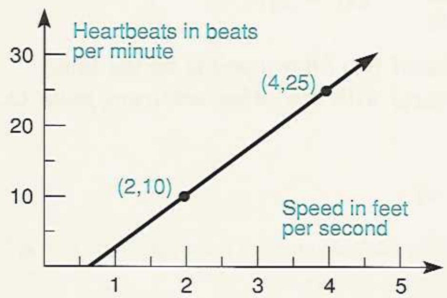
31. A ladder leaning against the side of a building touches the building at a point 12 meters from the ground. If the foot of the ladder is 18 meters from the base of the building, what is the slope of the ladder?

32. A guy wire is attached to a telephone pole. If the wire is attached to the ground at a point 15 feet from the base of the pole and to the pole at a point 10 feet up on the pole, what is the slope of the wire?

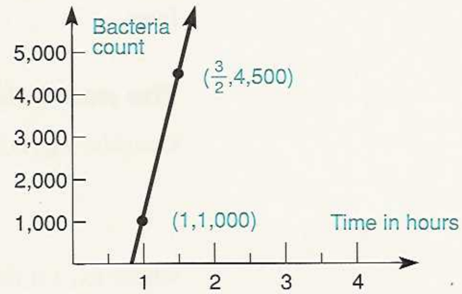
33. A company's profits (P) are related to the number of items produced (x) by a linear equation. If profits rise by \$1,000 for every 250 items produced, what is the slope of the graph of the equation?

34. A company's profits (P) are related to increases in a worker's average pay (x) by a linear equation. If the company's profits drop by \$1,500 per month for every increase of \$450 per year in the worker's average pay, what is the slope of the graph of the equation?

35. The diagram shows a linear representation of a jogger's heartbeat in beats per minute as his speed is increased (in feet per second). What is the slope of the line?



36. The diagram shows a linear representation of the bacteria count in a culture as related to the hours it exists. What is the slope of the line?



Review exercises

1. Simplify the expression $[3 - 4(5 - 2)]$. See section 1–6.

2. Evaluate $4^2 - 2^3 - 3 \cdot 5 - 2 \cdot 3$ See section 1–6.

Simplify the following. Use only positive exponents. Assume all variables are nonzero. See section 3–4.

3. $(-2)^{-3}$

4. $\dfrac{a^{-2}b^3}{a^3b^{-2}}$

5. $x^{-2} \cdot x^3 \cdot x^0$

6. When four times a number is increased by 12, the result is 64. What is the number? See section 2–8.

Solve the following equations for y. See section 2–6.

7. $3x + y = -1$

8. $3x - 3y = 6$

7–4 ■ The equation of a line

In section 7–2, we discussed the straight line graph of a linear equation in two variables. In this section, we discuss how to determine the equation of a straight line when we know certain facts about the line. There are three forms of the equation of a straight line that are of use to us: the *standard* form, the *point-slope* form, and the *slope-intercept* form.

The standard form

The **standard form** of the equation of a straight line is stated here.

Standard form of the equation of a line

$$ax + by = c$$

where a, b, and c are integers, $a \geq 0$, a and b not both zero.

To illustrate, the equations

$$2x + 3y = 6,\ 4x - 2y = 0, \text{ and } x - 6y = -2$$

are written in standard form. In most situations, equations will be written in this form.

The point-slope form

Consider again the definition for the slope of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad (x_1 \neq x_2)$$

where (x_1,y_1) and (x_2,y_2) are coordinates of two *known* points on the line. Suppose we replace the known point (x_2,y_2) with *any other arbitrary point* (x,y) on the line. Then the slope is given by

$$m = \frac{y - y_1}{x - x_1} \qquad (x_1 \neq x)$$

$$m(x - x_1) = y - y_1 \qquad \text{Multiply both members of the equation by } x - x_1$$

$$y - y_1 = m(x - x_1) \qquad \text{Symmetric property}$$

We call this the **point-slope form** of the equation of a line, where (x_1,y_1) is a known point on the line, m is the slope, and (x,y) is *any* other unknown point on the line.

Point-slope form of the equation of a line

$$y - y_1 = m(x - x_1)$$

where (x_1,y_1) is a known point on the line and m is the slope of the line.

We can use this form to find the equation of a line if we know the slope of the line and the coordinates of at least one point on the line.

■ Example 7–4 A

1. Find the standard form of the equation of a line having slope $m = 2$ and passing through the point $(4,-3)$.

   Using the point-slope form, we know $m = 2$ and $(x_1,y_1) = (4,-3)$.

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = 2(x - 4) \qquad \text{Replace } y_1 \text{ with } -3,\ x_1 \text{ with } 4, \text{ and } m \text{ with } 2$$

$$y + 3 = 2x - 8 \qquad \text{Multiply and subtract as indicated}$$

$$y = 2x - 11 \qquad \text{Add } -3 \text{ to both members}$$

$$-2x + y = -11 \qquad \text{Add } -2x \text{ to both members}$$

$$2x - y = 11 \qquad \text{Multiply each member by } -1$$

Note We wrote our final answer in *standard form, $ax + by = c$* where $a > 0$.

2. Find the standard form of the equation of the line passing through the points $(-3,2)$ and $(5,1)$.

We use the two points to find the slope and then choose one of the points to find the equation.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{(1) - (2)}{(5) - (-3)} \qquad \text{Replace } y_2 \text{ with 1, } y_1 \text{ with 2, } x_2 \text{ with 5, and } x_1 \text{ with } -3$$

$$m = \frac{-1}{8} = -\frac{1}{8}$$

Choosing *either* of the points $(-3,2)$ or $(5,1)$ together with the slope, we use the point-slope form.

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{8}[x - (-3)] \qquad \text{Use point } (-3,2). \text{ Replace } y_1 \text{ with 2, } x_1 \text{ with } -3, \text{ and } m \text{ with } -\frac{1}{8}$$

$$y - 2 = -\frac{1}{8}(x + 3) \qquad x - (-3) = x + 3$$

$$8(y - 2) = -1(x + 3) \qquad \text{Multiply each member by 8}$$

$$8y - 16 = -x - 3 \qquad \text{Perform indicated multiplications}$$

$$8y = -x + 13 \qquad \text{Add 16 to both members}$$

$$x + 8y = 13 \qquad \text{Add } x \text{ to each member}$$

Note If we had used the point $(5,1)$, then

$$y - y_1 = m(x - x_1)$$

$$y - 1 = -\frac{1}{8}(x - 5)$$

$$8(y - 1) = -1(x - 5)$$

$$8y - 8 = -x + 5$$

$$8y = -x + 13$$

$$x + 8y = 13 \qquad \text{(Produces the same equation.)}$$

▶ ***Quick check*** a. Find the equation of the line passing through $(-1,3)$ and having slope $m = -2$.

b. Find the equation of the line passing through the points $(-1,2)$ and $(3,4)$. ■

The slope-intercept form

Suppose a given line L, having slope m, passes through the point $(0,b)$, the y-intercept of the line. Using the point-slope form of the equation of a line,

$$y - b = m(x - 0) \qquad \text{Replace } y_1 \text{ with } b \text{ and } x_1 \text{ with 0}$$

Then $$y - b = mx \qquad x - 0 = x$$

and $$y = \underbrace{m}_{\text{Slope}}x + \underbrace{b}_{y\text{-intercept}} \qquad \text{Add } b \text{ to each member}$$

We call $y = mx + b$ the **slope-intercept** form of the equation of a line.

Slope-intercept form of the equation of a line

When a linear equation is stated in the form $y = mx + b$, m is the slope of the line and b is the y-intercept.

We can use the slope-intercept form of the equation of a line to find the slope and the y-intercept of a line and to graph a linear equation in two variables.

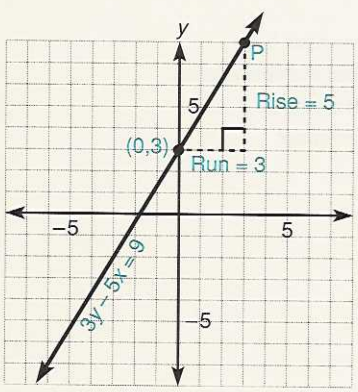
■ Example 7–4 B

1. Find the slope and the y-intercept of the line given by the equation $3y - 5x = 9$.

$$3y - 5x = 9 \qquad \text{Solve for } y$$
$$3y = 5x + 9 \qquad \text{Add } 5x \text{ to each member}$$
$$y = \frac{5x + 9}{3} \qquad \text{Divide each member by 3}$$
$$y = \frac{5}{3}x + 3 \qquad \text{Write in slope-intercept form by dividing 3 into 5 and into 9}$$

Then slope $m = \frac{5}{3}$ and y-intercept $b = 3$. [The y-intercept is the point (0,3).]

▶ ***Quick check*** Write the equation $3x + 4y = -12$ in slope-intercept form and determine the slope m and the y-intercept b.

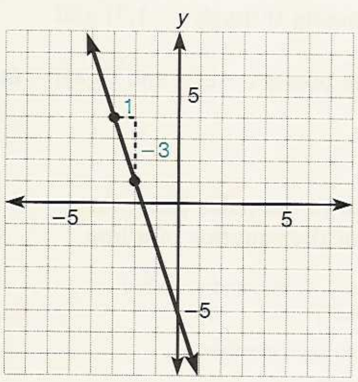


2. Sketch the graph of the equation $3y - 5x = 9$ using slope and y-intercept. From example 1, we determined the slope $m = \frac{5}{3}$ and the y-intercept $b = 3$. Then the point (0,3) is on the graph and we use the slope $\frac{5}{3}$ to find another point. To do this, we plot the y-intercept (0,3). Using the slope $\frac{5}{3}$, from the point (0,3) move 3 units to the *right* (the run, denominator) and from this point move 5 units *up* (the rise, numerator) to find a second point P. Draw a line through this point P and the y-intercept to obtain the graph.

3. Graph the line through $(-3,4)$ having slope $m = -3$. First, plot the point$(-3,4)$. Write the slope

$$m = -3 = \frac{-3}{1} = \frac{\text{negative rise (fall)}}{\text{run}}$$

From the point $(-3,4)$, move 1 unit right and then 3 units *down* (because of the negative sign) to find a second point P. Draw a line through point P and the given point $(-3,4)$.



▶ ***Quick check*** a. Graph the equation $3x + y = 2$ using the slope m and the y-intercept b.

b. Graph the line through $(-3,2)$ with slope $m = \frac{1}{3}$ ■

Given the slope of a line and its y-intercept, it is then possible to determine the equation of the line.

■ ***Example 7–4 C***

1. Find the equation of the line having slope $m = -3$ and y-intercept $b = 2$. Leave your answer in slope-intercept form.

Using $y = mx + b$

$y = (-3)x + (2)$ Replace m with -3 and b with 2

$y = -3x + 2$

2. Find the equation of the line having slope $m = \frac{2}{3}$ and passing through the point $(0,-4)$. Leave your answer in slope-intercept form.

Since the point $(0,-4)$ is on the y-axis, it is the y-intercept, so $b = -4$.

$y = mx + b$

$y = \left(\frac{2}{3}\right)x + (-4)$ Replace m with $\frac{2}{3}$ and b with -4

$y = \frac{2}{3}x - 4$

3. Find the equation of the line with slope $m = 0$ and having y-intercept 3.

$y = mx + b$

$y = (0)x + (3)$ Replace m with 0 and b with 3

$y = 0 + 3$

$y = 3$

Note The equation is of the form $y = b$ so the graph is a horizontal line.

4. Find the equation of the line with undefined slope and passing through $(3,-4)$.

Since m is undefined, the line is vertical and the equation is of the form $x = a$. Thus, $x = 3$ is the equation. ■

Parallel and perpendicular lines

Given two distinct straight lines in a plane, they will either be parallel (never meet no matter how far they are extended) or intersect in one point.

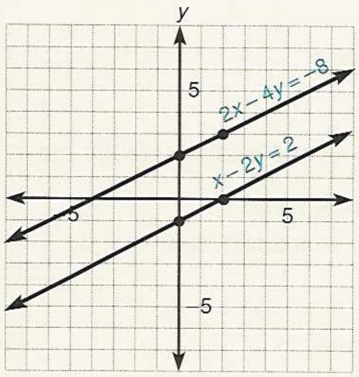
For two nonvertical lines to be parallel, they must *have the same slope and different y-intercepts.*

Slopes of parallel lines

Two distinct nonvertical lines having slopes m_1 and m_2 are parallel if and only if $m_1 = m_2$.

Note All vertical lines (whose slopes are undefined) are parallel to one another.

■ Example 7–4 D



Show that distinct lines $x - 2y = 2$ and $2x - 4y = -8$ are parallel lines.

We solve each equation for y to write them in slope-intercept form $y = mx + b$.

$$\begin{aligned} x - 2y &= 2 \\ -2y &= -x + 2 \\ y &= \frac{-x + 2}{-2} \\ y &= \frac{1}{2}x - 1 \\ m_1 &= \frac{1}{2} \end{aligned} \qquad \begin{aligned} 2x - 4y &= -8 \\ -4y &= -2x - 8 \\ y &= \frac{-2x - 8}{-4} \\ y &= \frac{1}{2}x + 2 \\ m_2 &= \frac{1}{2} \end{aligned}$$

Since $m_1 = \frac{1}{2} = m_2$, the two lines are parallel. ■

Two lines that intersect in a single point can be *perpendicular*. Perpendicular lines make *right angles* with one another. Two nonvertical lines having slopes m_1 and m_2, respectively, are perpendicular if and only if the *product* of their slopes is -1.

Slopes of perpendicular lines

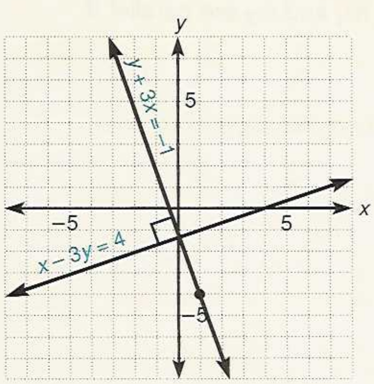
Two nonvertical lines having slopes m_1 and m_2 are perpendicular if and only if $m_1m_2 = -1$.

To illustrate, lines having slopes $\frac{3}{4}$ and $-\frac{4}{3}$ are perpendicular since

$$\left(\frac{3}{4}\right)\left(-\frac{4}{3}\right) = -1$$

Note The slopes $\frac{3}{4}$ and $-\frac{4}{3}$ are *negative reciprocals* of each other. From this, we can conclude that the slopes of perpendicular lines will be negative reciprocals.

■ Example 7–4 E



Show that the lines $x - 3y = 4$ and $3x + y = -1$ are perpendicular lines.

We solve for y to write each equation in slope-intercept form $y = mx + b$.

$$\begin{aligned} x - 3y &= 4 \\ -3y &= -x + 4 \\ y &= \frac{1}{3}x - \frac{4}{3} \\ m_1 &= \frac{1}{3} \end{aligned} \qquad \begin{aligned} 3x + y &= -1 \\ y &= -3x - 1 \\ m_2 &= -3 \end{aligned}$$

Since $m_1m_2 = \frac{1}{3} \cdot (-3) = -1$, the lines are perpendicular.

Note $\frac{1}{3}$ and -3 are *negative reciprocals* of each other. The box where the lines intersect indicates perpendicular lines.

▶ ***Quick check*** Find the slopes of the lines $3x + y = 4$ and $2x - y = 1$ and determine if the lines are parallel, perpendicular, or neither.

All vertical lines are perpendicular to all horizontal lines even though the product of their slopes does not exist. For example, the lines $x = -2$ and $y = 4$ are perpendicular.

Mastery points

Can you

- Write the equation of a line in standard form?
- Find the equation of a line knowing the slope and a point or two points on the line?
- Find the slope and y-intercept of a line knowing the equation of the line?
- Graph a linear equation in two variables using the slope and y-intercept?
- Graph a linear equation in two variables using the slope and a point of the line?
- Find the equation of a line given the slope and the y-intercept?
- Determine whether two lines are parallel or perpendicular?

Exercise 7–4

Write the equation of the line passing through the given point and having the given slope. See example 7–4 A–1, C.

Example $(-1,3)$; $m = -2$

Solution Use the *point-slope form* of the equation of a line.

$$
\begin{aligned}
y - y_1 &= m(x - x_1) \\
y - (3) &= (-2)[x - (-1)] && \text{Replace } y_1 \text{ with 3, } x_1 \text{ with } -1\text{, and } m \text{ with } -2 \\
y - 3 &= -2(x + 1) && \text{Definition of subtraction} \\
y - 3 &= -2x - 2 \\
y &= -2x + 1 \text{ or } 2x + y = 1 && \text{Written in standard form } ax + by = c
\end{aligned}
$$

$\boxed{1.}$ $(1,3)$; $m = 4$ **2.** $(-3,1)$; $m = 2$ **3.** $(0,-1)$; $m = -2$ **4.** $(0,4)$; $m = -3$

5. $(2,7)$; $m = \frac{3}{5}$ **6.** $(-1,5)$; $m = \frac{4}{7}$ **7.** $(5,0)$; $m = -\frac{7}{6}$ $\boxed{8.}$ $(-3,0)$; $m = -\frac{5}{8}$

9. $(-3,-7)$; $m = \frac{5}{4}$ **10.** $(-9,-2)$; $m = \frac{3}{2}$ **11.** $(2,-3)$; $m = 0$ **12.** $(-7,3)$; $m = 0$

13. $(1,-8)$; slope is undefined. **14.** $(0,4)$; slope is undefined.

Find the standard form of the equation of the line passing through each pair of given points. Write the equation in standard form $ax + by = c$, where a, b, and c are integers, $a \geq 0$. See example 7–4 A–2.

Example $(-1,2)$ and $(3,4)$

Solution Find the slope using

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{(4) - (2)}{(3) - (-1)}$$ Replace y_2 with 4, y_1 with 2, x_2 with 3, and x_1 with -1

$$= \frac{2}{4}$$ Subtract as indicated

$$= \frac{1}{2}$$

Use the point-slope form of the equation of a line and *one of the given points,* $(3,4)$.

$$y - y_1 = m(x - x_1)$$

$$y - (4) = \left(\frac{1}{2}\right)[x - (3)]$$ Replace y_1 with 4, x_1 with 3, and m with $\frac{1}{2}$

$$2y - 8 = x - 3$$ Mulitiply each member by 2 to clear denominator

$$2y = x + 5$$ Add 8 to each member

$$-x + 2y = 5$$ Subtract x from each member

$$x - 2y = -5$$ Multiply each member by -1

15. $(2,1)$ and $(6,3)$ **16.** $(3,2)$ and $(4,5)$ **17.** $(-3,2)$ and $(5,-1)$ **18.** $(-6,2)$ and $(4,-3)$

19. $(0,4)$ and $(-2,2)$ **20.** $(1,7)$ and $(0,-3)$ **21.** $(-6,0)$ and $(1,-1)$ **22.** $(5,6)$ and $(5,0)$

23. $(0,8)$ and $(-3,0)$ **24.** $(4,0)$ and $(0,-1)$ **25.** $(0,0)$ and $(-5,8)$ **26.** $(7,-1)$ and $(0,0)$

Write the following equations in slope–intercept form and determine the slope m and y-intercept b. See example 7–4 B–1.

Example $3x + 4y = -12$

Solution Solve for y.

$$3x + 4y = -12$$

$$4y = -3x - 12$$ Add $-3x$ to each member

$$y = \frac{-3x - 12}{4}$$ Divide each member by 4

$$y = -\frac{3}{4}x - 3$$ Divide -3 and -12 by 4

Then slope $m = -\frac{3}{4}$ and y-intercept $b = -3$

27. $x + y = 2$ **28.** $y - x = 3$ **29.** $3x + y = -2$ **30.** $y - 4x = 5$

31. $2x + 5y = 10$ **32.** $3x + 2y = 8$ **33.** $7x - 2y = -4$ **34.** $9x - 3y = 3$

35. $2y - 9x = -6$ **36.** $4y - 5x = -12$ **37.** $8x - 9y = 1$ **38.** $-7x + 4y = -5$

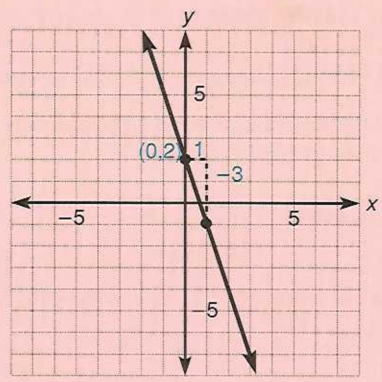
Use the slope m and the y-intercept b to graph the following equations. See example 7–4 B–2.

Example $3x + y = 2$

Solution Write the equation in slope-intercept form $y = mx + b$.

$y = -3x + 2$ so $m = -3$ and $b = 2$

Since $-3 = \frac{-3}{1}$, from the point (0,2), move 1 unit to the right and 3 units down to get a second point. Draw a straight line through the two points.

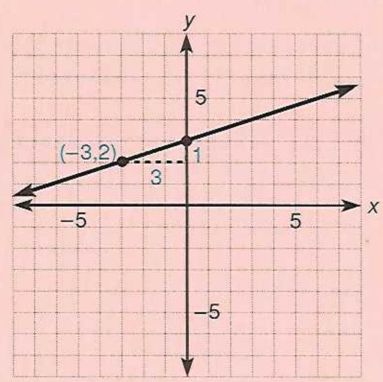


39. $y = 2x - 4$
40. $y = 3x + 1$
41. $y = -5x + 2$
42. $y = -2x - 3$
43. $y = \frac{2}{3}x - 1$
44. $y = \frac{-4}{3}x + 3$
45. $2x + y = -3$
46. $3x + y = -2$
47. $5x - 2y = 6$
48. $3x - 4y = 8$
49. $4x + 3y = -9$
50. $3x + 8y = -16$

Graph the line through the given point having the given slope m. See example 7–4 B–3.

Example $(-3,2)$; $m = \frac{1}{3}$

Solution Plot the point $(-3,2)$. Then move 3 units to the right and 1 unit up to find the second point. Draw a straight line through the two points.



51. $(4,3)$; $m = \frac{3}{4}$
52. $(-2,-2)$; $m = \frac{2}{3}$
53. $(-1,2)$; $m = \frac{-1}{3}$
54. $(2,-1)$; $m = \frac{-3}{2}$
55. $(-4,-5)$; $m = 4$
56. $(2,-3)$; $m = 2$
57. $(0,3)$; $m = -1$
58. $(1,5)$; $m = -3$

(*Hint:* For exercises 59 to 62, recall the types of lines that have slope $m = 0$ or undefined slope.)

59. $(5,6)$; $m = 0$
60. $(-3,1)$; $m = 0$
61. $(-1,-1)$; slope undefined
62. $(4,7)$; slope undefined

Find the equation of the line (in slope-intercept form) having the following characteristics. See Example 7–4 C.

63. Having slope $m = 4$ and y-intercept $b = -1$

64. Having slope $m = -8$ and y-intercept $b = 6$

65. Having slope $m = -\dfrac{5}{3}$ and passing through $(0,2)$

66. Having slope $m = \dfrac{3}{4}$ and passing through $(0,5)$

67. Having slope $m = 0$ and passing through $(0,-1)$

68. Having slope $m = 0$ and y-intercept4

Find the slope of each line and determine if each pair of lines is parallel, perpendicular, or neither. See example 7–4 D and E.

Example $3x + y = 4$
$2x - y = 1$

Solution Write each equation in slope-intercept form.

$$\begin{aligned} 3x + y &= 4 \\ y &= -3x + 4 \\ \text{so}\quad m_1 &= -3 \end{aligned} \qquad\qquad \begin{aligned} 2x - y &= 1 \\ y &= 2x - 1 \\ \text{so}\quad m_2 &= 2 \end{aligned}$$

Since $m_1 \neq m_2$, the lines are *not* parallel; and since $m_1m_2 = -3 \cdot 2 = -6$, the lines are not perpendicular. Therefore the lines are neither.

69. $x + y = 4$; $x + y = -7$

70. $-x + y = 9$; $-x + y = -2$

71. $x + y = 5$; $-x + y = -1$

72. $-x + y = 8$; $x + y = 3$

73. $3x - y = 2$; $6x + 2y = -5$

74. $x + 2y = 5$; $-6x + 3y = -1$

75. $5x + y = 1$; $10x + 3y = 2$

76. $8x - 9y = 1$; $16x + 7y = 3$

77. $4x - y = 5$; $12x - 3y = 1$

78. $4x + 6y = -3$; $6x + 9y = 4$

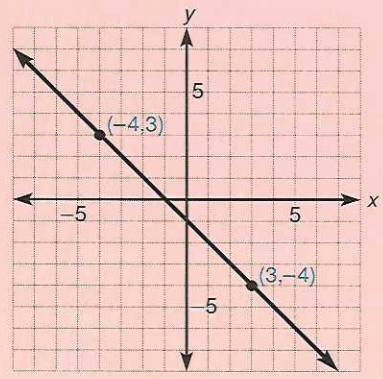
79. $y = 5$; $x = 2$

80. $x - 3y = -6$; $3x - 9y = -18$

Find the equation for the given line using the slope, one of the points, and the point-slope form $y - y_1 = m(x - x_1)$, where point (x_1,y_1) is a known point on the line.

Example It appears that the line passes through points $(-4,3)$ and $(3,-4)$.

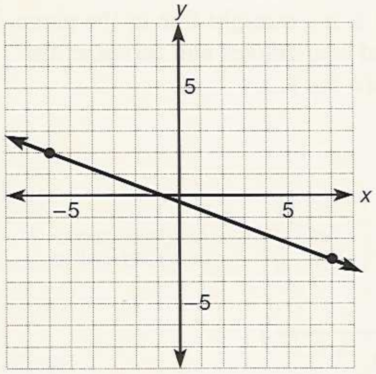
$$\text{Then } m = \frac{(3) - (-4)}{(-4) - (3)} = \frac{7}{-7} = -1$$



Use $y - y_1 = m(x - x_1)$ and the point $(-4,3)$.

$$\begin{aligned} y - (3) &= (-1)[x - (-4)] \\ y - 3 &= -1(x + 4) \\ y - 3 &= -x - 4 \\ y &= -x - 1 \\ x + y &= -1 \end{aligned}$$

81.



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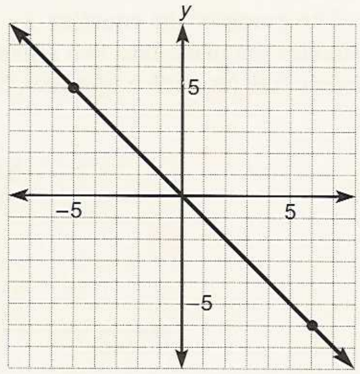

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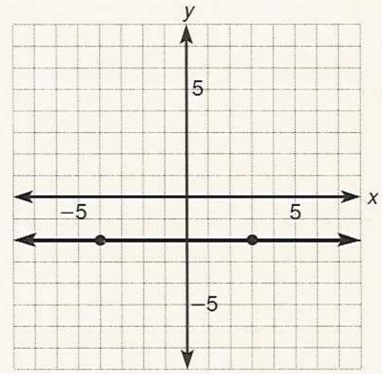

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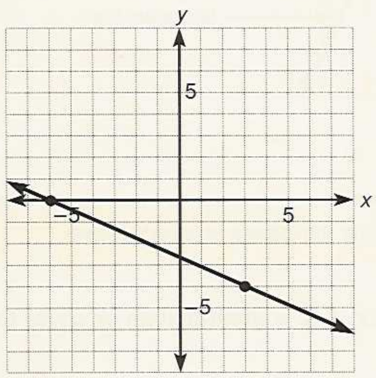

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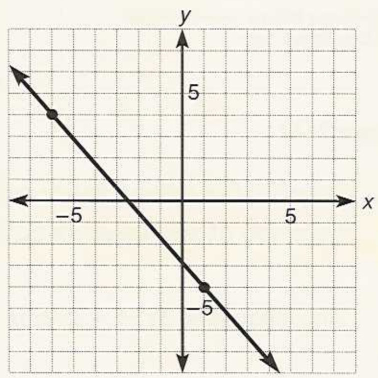
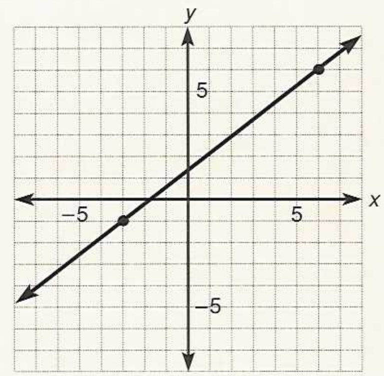


86.



Review exercises

1. A 100-foot extension cord is to be cut into two pieces so that one piece is 17 feet longer than the other. What is the length of each piece? See section 2–8.

2. From the numbers $0, -3, \frac{3}{4}, 9, -\frac{7}{8}, \frac{9}{3}$, and $\frac{0}{4}$, choose the integers. See section 1–3.

Find the solution set of the following equations. See section 2–6.

3. $4y + 1 = 2y - 5$ **4.** $3(2y - 1) = -1(y + 2)$ **5.** $\frac{3x}{5} - \frac{x}{2} = 1$

Factor the following expressions. See sections 4–2, 4–3, and 4–4. If the expression does not factor, so state.

6. $3x^2 + 4x - 4$ **7.** $x^2 + 5x + 7$ **8.** $8y^2 - 32x^2$

7–5 ■ Graphing linear inequalities in two variables

In section 7–2, we graphed linear equations in two variables such as $2x + y = 6$. In this section, we consider the graphs of *linear inequalities* in two variables such as

$$2x + y < 6 \quad \text{and} \quad 2x + y \geq 6$$

where the equals to sign has been replaced by one of the four inequality symbols

$$<, \leq, >, \text{or} \geq$$

The graph of the equation $2x + y = 6$ is a straight line in the plane. This line divides the plane into two regions called *half-planes* and serves as the *boundary line* for each half-plane. See figure 7–11.

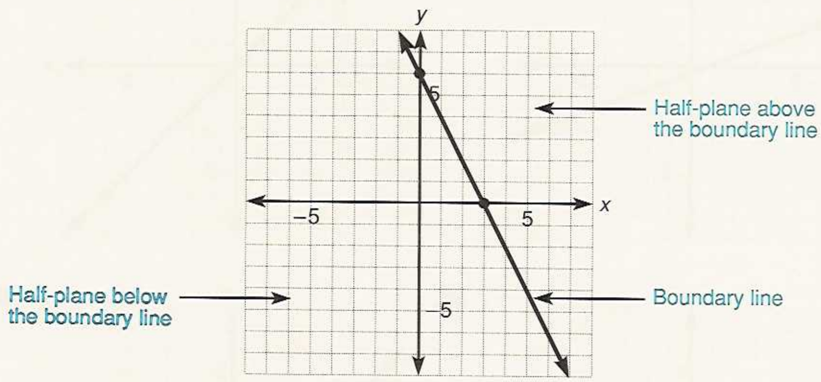


Figure 7–11

Graph of a linear inequality

The graph of any linear inequality in two variables is a half-plane.

The inequality $2x + y \geq 6$ is read "$2x + y$ is greater than or equal to 6" So,

$$2x + y > 6 \quad \text{or} \quad 2x + y = 6$$

The graph *will consist of the boundary line* and the proper half-plane. The inequality $2x + y > 6$ is read "$2x + y$ is greater than 6" so the boundary line *is not* a part of the graph of the inequality. To indicate this, the boundary line is drawn as a dashed line rather than a solid line.

1. For $2x + y \geq 6$, the boundary line $2x + y = 6$ is a *solid* line, to show that the points on the line are included.
2. For $2x + y > 6$, the boundary line $2x + y = 6$ is a *dashed* line to show that the points on the line are *not* included.

See figure 7–12.

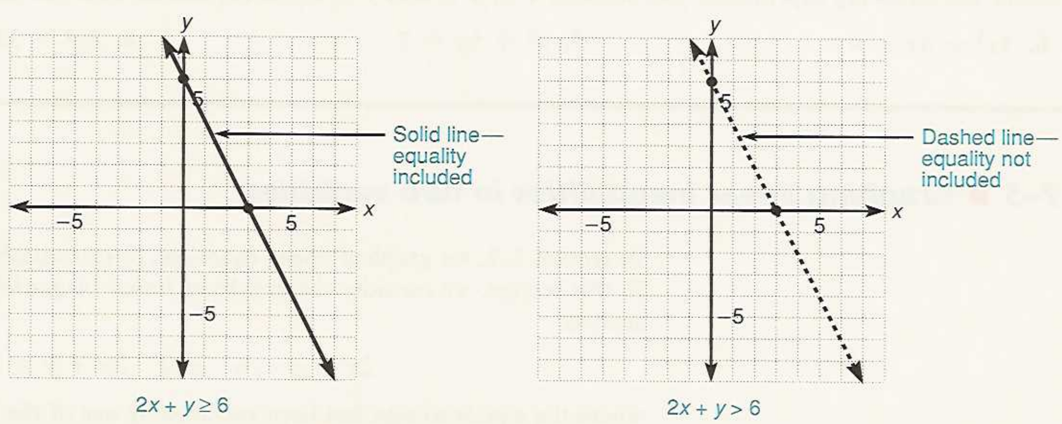


Figure 7–12

To determine the proper half-plane to shade, we choose a *test point* in one of the half-planes [usually the origin, (0,0)] and substitute into the inequality.

1. If the test point satisfies the inequality, shade the half-plane containing that point,
2. If the test point *does not satisfy* the inequality, shade the half-plane that does not contain the point.

To illustrate, consider the inequality $2x + y > 6$ and the test point (0,0).

$$2x + y > 6$$
$$2(0) + (0) > 6 \quad \text{Replace } x \text{ with 0 and } y \text{ with 0}$$
$$0 + 0 > 6$$
$$0 > 6 \quad \text{(False statement)}$$

Shade the half-plane that *does not* contain the origin (0,0). See figure 7–13(a).

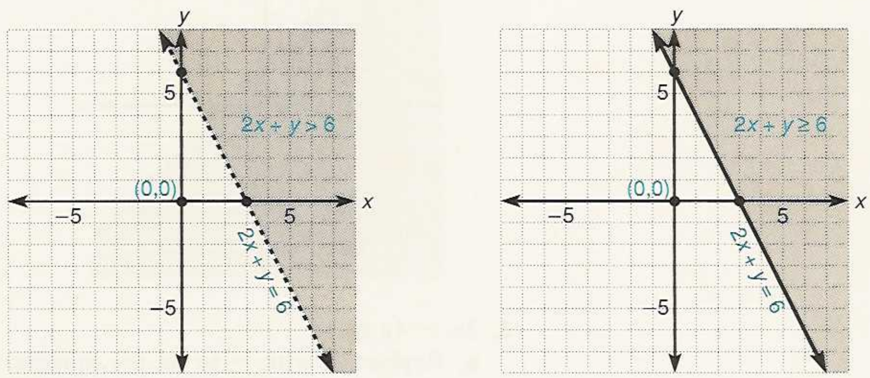


Figure 7–13

Note We will use the origin (0,0) as the test point in all cases except when the boundary line passes through the origin.

Graphing a linear inequality in two variables

1. Replace the inequality symbol with the equality symbol.
2. Graph the resulting equation for the boundary line which will be a
   a. *solid line* if the inequality symbol is $\leq$ or $\geq$,
   b. *dashed line* if the inequality symbol is $<$ or $>$.
3. Choose some test point that is *not* on the line [usually the origin (0,0)] and substitute the coordinates of the point into the inequality.
4. If the resulting statement is
   a. *true,* shade the half-plane containing the test point,
   b. *false,* shade the half-plane that *does not* contain the test point.

■ Example 7–5 A

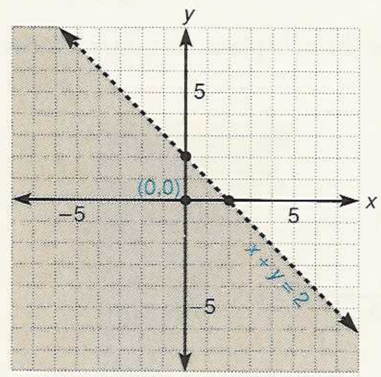
Graph the following linear inequalities.

1. $x + y < 2$

a. Replace $<$ with $=$ to get the equation $x + y = 2$.
b. Graph $x + y = 2$ as a *dashed* line since we have $<$.
c. Since the boundary line does not go through the origin, choose test point (0,0).

$$\begin{aligned} x + y &< 2 \\ (0) + (0) &< 2 \quad \text{Replace } x \text{ with 0 and } y \text{ with 0} \\ 0 &< 2 \quad \text{(True)} \end{aligned}$$

d. Shade the half-plane containing the origin (0,0).

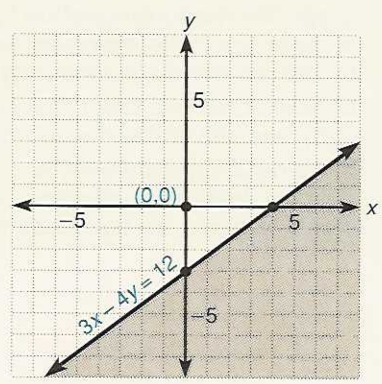


2. $3x - 4y \geq 12$

a. Replace $\geq$ with $=$ to get the equation $3x - 4y = 12$.
b. Graph $3x - 4y = 12$ as a *solid* line since we have $\geq$.
c. Choose test point (0,0).

$$\begin{aligned} 3x - 4y &\geq 12 \\ 3(0) - 4(0) &\geq 12 \\ 0 &\geq 12 \quad \text{(False)} \end{aligned}$$

d. Shade the half-plane that *does not* contain the origin (0,0).



3. $y \le -2$

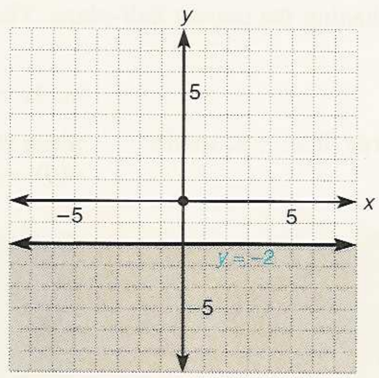
a. Replace $\le$ with $=$ to get the equation $y = -2$.

b. Graph $y = -2$ as a *solid* horizontal line since we have $\le$.

c. Choose the test point (0,0).

$$y \le -2$$
$$0 \le -2 \quad \text{(False)}$$

d. Shade the half-plane that *does not* contain the origin.



4. $3y - 2x > 0$

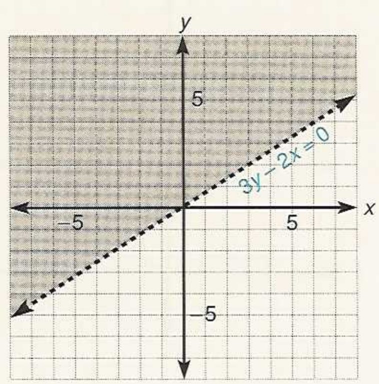
a. Replace $>$ with $=$ to get the equation $3y - 2x = 0$.

b. Graph $3y - 2x = 0$ as a *dashed* line since we have the inequality $>$.

c. Since the graph goes through the origin, choose another test point not on the line, say (2,4).

$$3y - 2x > 0$$
$$3(4) - 2(2) > 0 \quad \text{Replace } x \text{ with 2 and } y \text{ with 4}$$
$$12 - 4 > 0$$
$$8 > 0 \quad \text{(True)}$$

d. Shade the half-plane containing the point (2,4).



▶ ***Quick check*** Graph the linear inequality. $2y + x < 5$ ■

Mastery points

Can you

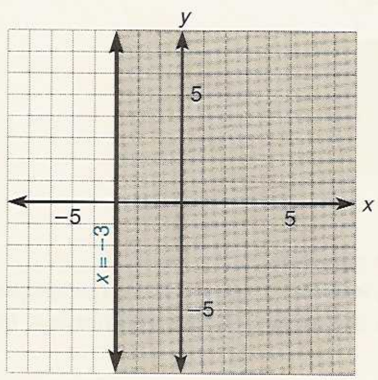
- Graph the solution set of a linear inequality in two variables?
- Distinguish when the boundary line is solid and when it is dashed?

Exercise 7–5

Complete the graph of the inequality by shading the correct half-plane. The correct boundary line has been drawn. See example 7–5 A.

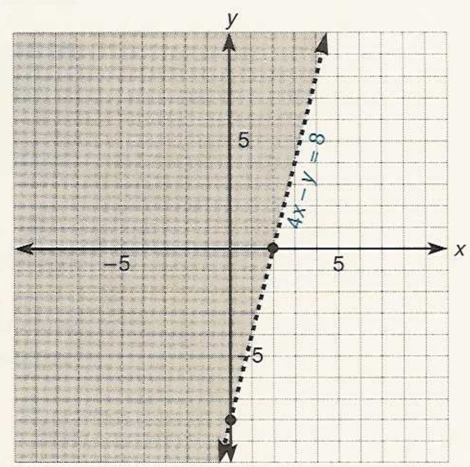
Examples a. $x \geq -3$

Solutions Since (0,0) makes $x \geq -3$ true, $0 \geq -3$, shade to the right of $x = -3$.

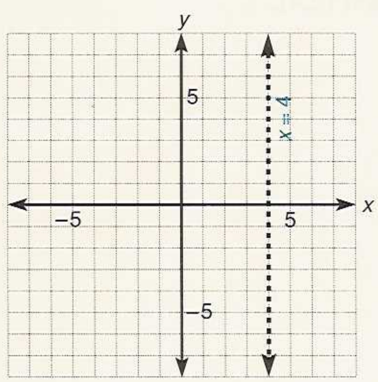


b. $4x - y < 8$

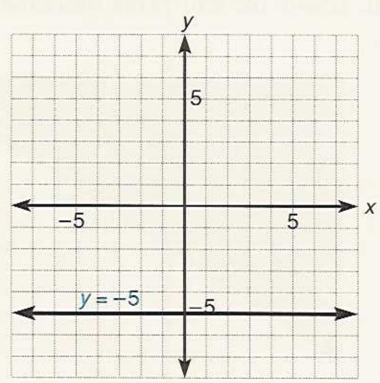
Since (0,0) makes $4x - y < 8$ true, $4(0) - 0 < 8$, shade to the left of $4x - y = 8$.



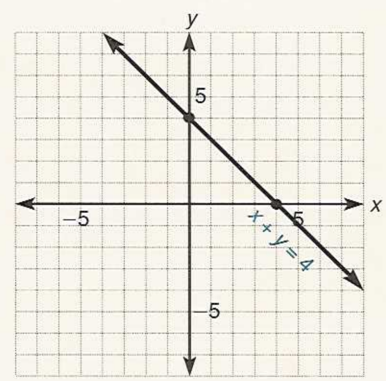
1. $x < 4$



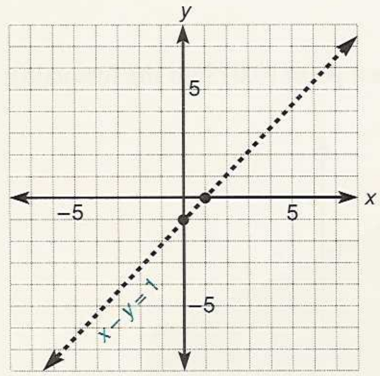
2. $y \geq -5$



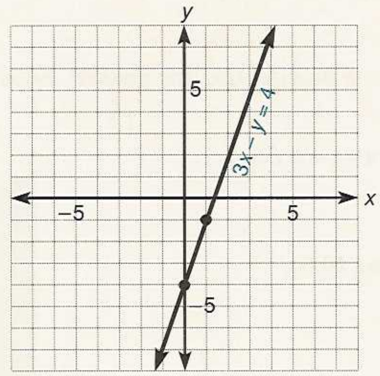
3. $x + y \geq 4$



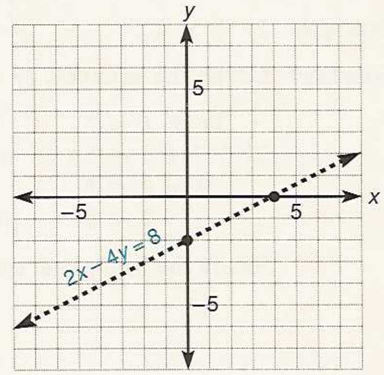
4. $x - y < 1$



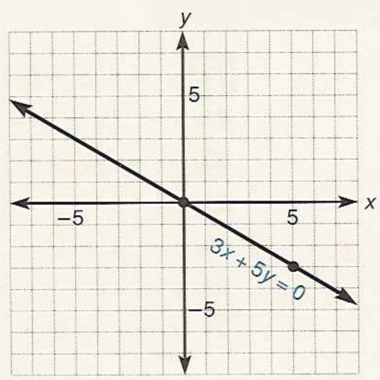
5. $3x - y \leq 4$



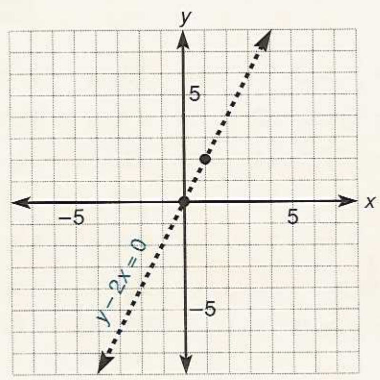
6. $2x - 4y > 8$



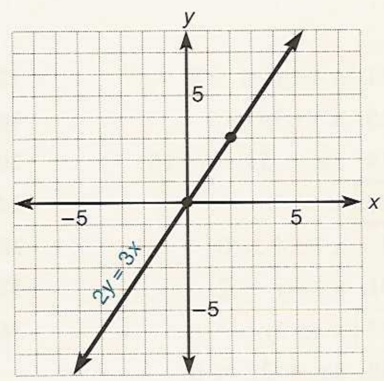
7. $3x + 5y \geq 0$



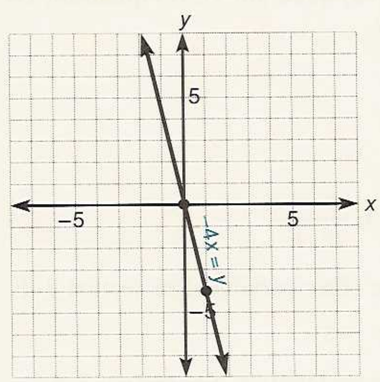
8. $y - 2x < 0$



9. $2y \leq 3x$



10. $-4x \geq y$



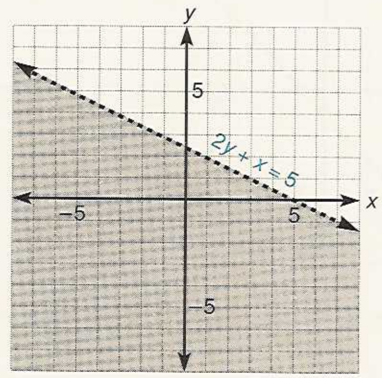
Graph the given linear inequality. See example 7–5 A.

Example $2y + x < 5$

Solution Graph the equation $2y + x = 5$ making the line *dashed*. Substitute test point (0,0) into the inequality.

$$2y + x < 5$$
$$2(0) + (0) < 5 \quad \text{Replace } x \text{ by 0 and } y \text{ by 0}$$
$$0 + 0 < 5$$
$$0 < 5 \quad \text{(True)}$$

Shade the half-plane containing (0,0).



11. $x + y \leq -2$
12. $x + y > 3$
13. $x - y < 6$
$\boxed{14.}$ $x - y \geq -5$
15. $x + 3y > 1$
16. $x + 2y \leq -2$
17. $y \geq 4x + 1$
18. $y < 2x - 3$
19. $3x - 5y < -15$
20. $2x - 7y > 14$
21. $5x - 4y \leq 20$
22. $5x + 2y \leq -10$
$\boxed{23.}$ $x > 3$
24. $x > -4$
25. $x \leq -2$
26. $x < 5$
27. $x + 4 > 0$
28. $x + 5 \leq -2$
29. $y + 3 \leq 5$
30. $y - 2 > 1$
31. $y > x - 2$
32. $3x \geq y$
33. $x < -y$
34. $y \leq 4x$
$\boxed{35.}$ $x \geq 3y$
36. $x + y \geq 0$
37. $8x - 3y \geq 0$
38. $x - y < 0$
39. $3x + 2y < 0$
40. Write an inequality describing all points in the plane that lie (a) below the x-axis, (b) to the right of the y-axis.
41. Write inequalities describing all points that lie in (a) quadrant I, (b) quadrant II, (c) quadrant III, (d) quadrant IV.

Review exercises

Find the domain of the following rational expressions. See section 5–1.

1. $\dfrac{2x + 1}{x - 3}$
2. $\dfrac{y - 4}{y^2 + 3y - 10}$
3. $\dfrac{4 - x}{x^2 + 1}$

Find the standard form of the value of the unknown variable given the value of the other variable. See section 7–1.

4. $4x - 2y = 3; x = 2$
5. $4y + x = 1; y = -3$

Find the standard form of the equation of the straight line having the following characteristics. See section 7–4.

6. Through points $(-2,3)$ and $(4,1)$
7. Having slope $\dfrac{3}{4}$ and y-intercept -2.
8. Vertical line through $(-3,6)$
9. Horizontal line through $(8,-3)$

7–6 ■ Functions defined by linear equations in two variables

Functions

In section 7–1, we studied equations in two variables such as

$$3x + 2y = 4 \quad \text{or} \quad y = 4 - 3x$$

which related values for x and y that satisfied the given equation. We showed this relationship by ordered pairs (x,y), where, in *most* cases, the value of y was dependent on a chosen value for x.

In all phases of mathematics, from the most elementary to the most sophisticated, the idea of a function is a cornerstone for each mathematical development. Phrases such as "price is a function of cost" or "price is determined by the cost to manufacture," in the business world, and "velocity is a function of time" or "velocity depends on time," of a falling object in science, are common. What is being said in each case is that "the price of an item will change when the cost of producing the item changes" and "the velocity of a falling object will change as the falling time changes."

In mathematics, a function is a special kind of relation, so we first define a **relation.**

Relation

A **relation** is any set of ordered pairs.

Thus, the set of ordered pairs $\{(1,2), (-3,6), (0,-5), (3,4)\}$ is a relation. Every relation has a **domain** and a **range.**

Domain and range of a relation

1. The **domain** of a relation is the set of all first components of the ordered pairs.
2. The **range** of a relation is the set of all second components of the ordered pairs.

Thus, in the relation $\{(1,2), (-3,6), (0,-5), (3,4)\}$,

$$\text{the domain} = \{1,-3,0,3\} \text{ and the range} = \{2,6,-5,4\}$$

The correspondence between the elements of the domain and the elements of the range is shown in figure 7–14.

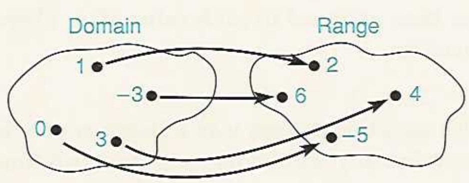


Figure 7–14

Recall, we initially said that a function is a special relation—a set of ordered pairs with a special characteristic.

Function

A **function** is a relation in which no two distinct ordered pairs have the same first component.

From this definition, each member of the domain is assigned to one and only one member of the range.

Example 7–6 A

Determine whether the following relations are functions.

1. $\{(1,2), (-3,6), (0,-5), (3,4)\}$
   No two distinct ordered pairs have the same first component, so the relation is a function.
2. $\{(-3,5), (2,4), (2,-6), (-7,0)\}$
   Since the ordered pairs $(2,4)$ and $(2,-6)$ have the same first component, this *is not* a function.
3. $\{(1,3), (-2,3), (0,3), (4,3)\}$
   Even though the second component of every ordered pair is the same number, this does not violate the definition. This relation is a function. ■

Many useful functions have infinitely many ordered pairs and must be defined by an equation that tells how to get the second component when we are given the first component.

Usually we use x and y to denote the variables in a mathematical function. By our definition, for *y to be a function of x*, the two variables *must be related* so that for each value assigned to x, there is a unique (one and only one) corresponding value assigned to y. This is a very important concept for us to remember. When we use an equation to determine the outcome in a given situation, we want our equation to be of the type that gives us only one result.

Consider the following equations involving the two variables x and y.

Example 7–6 B

1. $y = x + 1$
   This equation defines y as a function of x because, for each chosen value of x, the value of y is one more than the value of x. Thus the ordered pairs $(1,2)$, $(-3,-2)$, $\left(\frac{1}{3},\frac{4}{3}\right)$, and so on, all satisfy this equation. *Only one* value of y has been assigned to each value of x. Therefore, $y = x + 1$ defines a function.
2. $y = 2x - 1$
   This equation defines y as a function of x because, for each chosen value of x, the value of y will be one less than two times x. The ordered pairs $(2,3)$, $(-3,-7)$, $(0,-1)$, $\left(\frac{3}{2},2\right)$, and so on, all satisfy this equation. Only one value of y is associated with each value of x. Therefore, $y = 2x - 1$ defines a function.

3. $x = -4$
This equation *does not* define y as a function of x. Recall that in section 7–2 we found this equation defines a set of ordered pairs whose first components are all -4. Thus while x can only have the value -4, y can take on values 3 and -5 (as well as many other values). The two distinct ordered pairs, $(-4,3)$ and $(-4,-5)$, have the *same first component* and are solutions of this equation. Thus $x = -4$ does not define a function.

4. $y = -3$
This equation *does* define a function since it can be written $y = 0 \cdot x - 3$ and for *any* real value of x, y is determined to be -3. That is, $(2,-3)$, $(-1,-3)$, $(0,-3)$, and so on, are all solutions, and for each value of x, there is only *one* value of y.

Note The function defined by $y = a$ is often called the "constant function." ■

From the definitions, we see that if y is defined to be a function of x, then x represents an element in the *domain* and y represents an element in the *range* of the function. The domain of a function must always be stated, or implied by the nature of the function. In this text, if the domain is not stated, we assume it to be the *set of all real numbers* for which y is defined or makes sense.

■ Example 7–6 C

Determine the domain and range for the function defined by

$2x + y = 1$, where $x \in \{-4,-2,0,2,4\}$.

The domain is restricted to be the set $\{-4,-2,0,2,4\}$, even though x could take on infinitely many values.

Since we are given values of x, we will find it easier to solve for y *once* and then substitute the values of x. Otherwise, we will have to solve for y *each time* there is a new value of x. Solving for y, $y = 1 - 2x$. When

$x = -4$, then $y = 1 - 2(-4) = 1 + 8 = 9$
$x = -2$, then $y = 1 - 2(-2) = 1 + 4 = 5$
$x = 0$, then $y = 1 - 2(0) = 1 - 0 = 1$
$x = 2$, then $y = 1 - 2(2) = 1 - 4 = -3$
$x = 4$, then $y = 1 - 2(4) = 1 - 8 = -7$

The range is the set $\{9,5,1,-3,-7\}$.
The function defined by the equation $2x + y = 1$ and the given restricted domain is represented by the set of ordered pairs $\{(-4,9), (-2,5), (0,1), (2,-3), (4,-7)\}$.

▶ ***Quick check*** State the domain and range of the function defined by the equation $3x = 2y + 1$; $x \in \{1,3,5,7\}$. ■

We use the letters f, g, and h to name functions. For example, the function defined by the equation $y = 5x + 2$ is often written

$$f(x) = 5x + 2$$

Read the symbol "$f(x)$" as "f at x" or "f of x," which means "the value of the function f at x."

Note If we call a function f, then $f(x)$ means y in equations. Since the set of values of y is called its range, all values of $f(x)$ are also in the range.

■ Example 7–6 D

In the function f defined by $f(x) = 5x + 2$, if $x = 3$, find $f(3)$.

$$\begin{aligned} f(3) &= 5(3) + 2 \quad \text{Replace } x \text{ with 3 in } f(x) = 5x + 2 \\ &= 15 + 2 \\ &= 17 \end{aligned}$$

Thus $f(3) = 17$, which means the value of the function f is 17 when $x = 3$. In like fashion, when

1. $x = -2$, then $f(-2) = 5(-2) + 2$ Replace x with -2
$$\begin{aligned} &= -10 + 2 \\ &= -8 \end{aligned}$$
Thus $f(-2) = -8$.

2. $x = \dfrac{1}{5}$, then $f\left(\dfrac{1}{5}\right) = 5\left(\dfrac{1}{5}\right) + 2$ Replace x with $\dfrac{1}{5}$
$$\begin{aligned} &= 1 + 2 \\ &= 3 \end{aligned}$$
Thus $f\left(\dfrac{1}{5}\right) = 3$.

3. $x = 0$, then $f(0) = 5(0) + 2$ Replace x with 0
$$\begin{aligned} &= 0 + 2 \\ &= 2 \end{aligned}$$
Thus $f(0) = 2$.

We have just determined that the function f defined by $f(x) = 5x + 2$ contains the ordered pairs $(3,17)$, $(-2,-8)$, $\left(\dfrac{1}{5},3\right)$, and $(0,2)$. There *are* many more ordered pairs in function f.

▶ ***Quick check*** a. Given $f(x) = 3x - 2$, find $f(-3)$, $f(0)$, and $f(4)$.
b. Given $f(x) = 5$, find $f(2)$, $f(-3)$, and $f(0)$.
c. Given $f(x) = 2 - 4x$, find $f(-1)$, $f(0)$, and $f(1)$ and state the answers as second components of ordered pairs belonging to the function. ■

When discussing arbitrary functions involving the variables x and y, we often see the statement

$$y = f(x)$$

This states that "y is a function of x."

Functions may be defined by many equations other than linear equations. In chapter 10, we will discuss another type of equation that defines a function.

Mastery points

Can you

- Determine the domain and range of a function?
- Understand what $f(x)$ means and find particular values of $f(x)$?

Exercise 7–6

State the domain and range of each of the functions defined by the equation using the restricted values of x. See example 7–6 C.

Example $3x = 2y + 1;\ x \in \{1,3,5,7\}$

Solution The domain $= \{1,3,5,7\}$.
Solving for y:

$$2y + 1 = 3x$$
$$2y = 3x - 1$$
$$y = \frac{3x - 1}{2}$$

When $x = 1,\ y = \dfrac{3(1) - 1}{2} = \dfrac{2}{2} = 1$

$x = 3,\ y = \dfrac{3(3) - 1}{2} = \dfrac{8}{2} = 4$

$x = 5,\ y = \dfrac{3(5) - 1}{2} = \dfrac{14}{2} = 7$

$x = 7,\ y = \dfrac{3(7) - 1}{2} = \dfrac{20}{2} = 10$

The range $= \{1,4,7,10\}$.

1. $y = 3x - 1;\ x \in \{-5,-3,-1,0,1,3,5\}$

2. $y = 2 - x;\ x \in \{-4,-2,0,2,4\}$

3. $2x - 3y = 1;\ x \in \{0,1,2,3,4,5\}$

4. $5x + y = 3;\ x \in \{-4,-3,-2,-1,0\}$

5. $y = -4;\ x \in \{-10,-5,0,5,10\}$

6. $y = 12;\ x \in \{-8,-4,-2,0,2,4,8\}$

Find the indicated value of the given function. See example 7–6 D.

Examples a. $f(x) = 3x - 2$; find $f(-3), f(0), f(4)$.

b. $f(x) = 5$; find $f(2), f(-3), f(0)$.
Rewrite $f(x) = 5$ as $f(x) = 0 \cdot x + 5$

Solutions a. $f(-3) = 3(-3) - 2 = -9 - 2 = -11$
$f(0) = 3(0) - 2 = 0 - 2 = -2$
$f(4) = 3(4) - 2 = 12 - 2 = 10$

$f(-3) = -11; f(0) = -2; f(4) = 10$

b. $f(2) = 0 \cdot 2 + 5 = 5$
$f(-3) = 0 \cdot -3 + 5 = 5$
$f(0) = 0 \cdot 0 + 5 = 5$

$f(2) = 5; f(-3) = 5; f(0) = 5$

7. $f(x) = 2x; f(2), f(0), f(-1)$

8. $f(x) = 3x; f(3), f(0), f(-4.3)$

9. $f(t) = t + 5; f(5), f(-5), f(0)$

10. $f(x) = x - 4; f(4), f(2), f(5.2)$

11. $f(x) = 3x + 5; f(-3), f\left(-\frac{5}{3}\right), f(1)$

12. $f(s) = 5s + 2; f(-2), f(3), f\left(-\frac{2}{5}\right)$

13. $f(x) = 3; f(-7), f(0), f(0.41)$

14. $f(x) = -5; f(-5), f(9), f(0)$

Find the indicated value of the given function. State the answers as second components of ordered pairs belonging to the function. See example 7–6 D.

Example $f(x) = 2 - 4x$; find $f(-1), f(0), f(1)$.

Solution $f(-1) = 2 - 4(-1) = 2 + 4 = 6$ Answer: $(-1,6)$
$f(0) = 2 - 4(0) = 2 - 0 = 2$ Answer: $(0,2)$
$f(1) = 2 - 4(1) = 2 - 4 = -2$ Answer: $(1,-2)$

15. $f(x) = 5x + 1; f(-3), f\left(-\frac{2}{5}\right), f(0), f(4), f\left(\frac{1}{5}\right)$

16. $f(x) = 7 - 2x; f(-4), f\left(-\frac{1}{2}\right), f(0), f\left(\frac{1}{2}\right), f(1.7)$

17. $f(x) = \frac{2}{3}x + 1; f(6), f(-3), f(0), f\left(\frac{3}{2}\right), f\left(-\frac{3}{2}\right)$

18. $f(x) = 5 - \frac{1}{4}x; f(-8), f\left(-\frac{4}{3}\right), f(0), f(4), f(0.13)$

19. $f(x) = 0; f(-1), f(-10), f(0), f\left(\frac{8}{3}\right), f(2)$

20. $f(x) = -8; f(-3), f\left(-\frac{3}{2}\right), f(0), f\left(\frac{5}{7}\right), f(14.7)$

Solve the following word problems.

21. The temperature in degrees Fahrenheit (F) can be expressed as a function of degrees Celsius (C) by $F = f(C)$. If $f(C) = \frac{9}{5}C + 32$, find $f(100), f(0), f(25), f(-10)$.

22. The temperature in degrees Celsius (C) can be expressed as a function of degrees Fahrenheit (F) by $C = g(F)$. If $g(F) = \frac{5}{9}(F - 32)$, find $g(32)$, $g(212)$, $g(-4)$, $g(59)$.

23. The cost c in cents of sending a letter by first-class mail is a function of the weight w in ounces of the letter defined by $c = 29w$. Then $c = h(w) = 29w$. Find $h(2)$, $h(3)$, $h(4)$. What is the domain of function h?

24. The perimeter P of a square can be expressed as a function of its sides s. If $P = 4s$, then $P = f(s) = 4s$. Find $f(4), f\left(\frac{3}{4}\right), f(4.7), f\left(\frac{5}{2}\right)$. What is the domain of f?

25. The resistance R in an electric circuit can be expressed as a function of the voltage E. If $R = 0.07E$, then $R = g(E) = 0.07E$. Find $g(2)$, $g(3)$, $g(1.2)$.

26. When two objects are pressed together, the frictional force F_f between their surfaces is related to the perpendicular (or normal) force N holding the surfaces together by $F_f = \mu N$, where μ is a constant coefficient of friction. For wood on wood, the coefficient of static friction is $\mu = 0.5$. Using h, define F_f as a function of N and find $h(100)$, $h(50)$, $h(25)$.

27. Using exercise 26, for a rubber tire on dry concrete, the coefficient of sliding friction is $\mu = 0.7$. Define a function g and find $g(5)$, $g(10)$, $g(3)$.

28. The circumference C of a circle may be expressed as a function of its radius r. Given $C = 2\pi r$, where 2π is a constant, express C as a function of r using f and find $f(7)$, $f(0.8)$, and $f\left(\frac{5}{2}\right)$. State the domain of function f.

29. The temperature F (in degrees Fahrenheit) can be approximated by the formula

$$F = \frac{n}{4} + 40$$

where n is the number of times a cricket chirps in 1 minute. Express F as a function of n using h to name the function and find $h(50)$, $h(12)$, $h(120)$.

30. Simple interest, I, on a loan for one year at 12% interest is given by $I = (.12)P$, where P is the amount borrowed. Using f to name the function, express I as a function of P and find $f(1{,}000)$, $f(5{,}000)$, $f(12{,}350)$.

31. The gross wages W of an hourly worker are determined by
$W =$ hourly rate (r) $\times$ hours worked (h) or
$W = rh$.
Define W as a function of h when $r = \$10.50$. Calling this function g, find $g(40)$, $g(48)$, $g(28)$.

32. A company's income statement, I, for one quarter is given by $I = R - E$, where R is the revenue in sales and gains and E is the expenses incurred. Define $I = h(R,E) = R - E$ and find $h(1{,}000;350)$, $h(875;490)$, $h(2{,}515;1{,}031)$. (*Hint:* $h(1{,}000;350)$ means that $R = \$1{,}000$ and $E = 350$.)

33. The selling price, S, of some merchandise is given by $S = C + M$, where C is the cost price of the merchandise and M is the markup (expenses incurred in operations). Using f, define S as a function of C and M; that is $S = f(C,M) = C + M$. Find $f(25,7)$, $f(32,11)$, $f(146,27)$.

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Review exercises

Solve the following equations for y. See section 2–7.

1. $2x + y = 7$
2. $3x - 4y = 8$
3. Given $x = 4y - 2$ and $3y - 2x = 1$, replace x with $4y - 2$ in the second equation and solve for y. See section 2–2.
4. Graph the equations $3x - 2y = 6$ and $x + y = 2$ on the same set of axes. Determine the coordinates of the point of intersection. See section 7–2.

Find the value of the unknown variable in each of the following equations. See section 7–1.

5. $x + 3y = 4$ when $y = -2$
6. $4y - 5x = 3$ when $x = 1$

Chapter 7 lead-in problem

Tickets to a football game at Smith High School cost \$1.25 for students and \$3.00 for adults. The total receipts for a game with University High School were \$1,020. Write an equation using x for the number of students and y for the number of adults attending the game.

Solution

Let x = the number of students at the game
y = the number of adults at the game

	Cost per ticket	Number	Receipts
Students	1.25	x	$1.25x$
Adults	3.00	y	$3.00y$

We add the individual receipts from the students and the adults to obtain the total receipts, \$1,020.

$$1.25x + 3.00y = 1{,}020$$

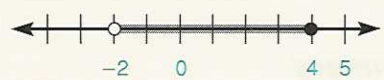
Chapter 7 summary

1. A **linear equation in two variables** is an equation that can be written in the form $ax + by = c$, where a, b, and c are real numbers, and a and b are not both zero.
2. The **solutions** of a linear equation in two variables are the ordered pairs of numbers of the form (x,y) that satisfy the equation.
3. Solutions of linear equations in two variables are found by choosing a value for one variable and evaluating to find the value of the other variable.
4. **Ordered pairs of numbers** may be graphed as points in the **rectangular coordinate plane.**
5. In the ordered pair (x,y), we call x the **abscissa** and y the **ordinate** of the point in the plane that is its graph.
6. Graphs of linear equations in two variables are *straight lines*.
7. To find the *x-intercept*, the abscissa of the point where the graph crosses the x-axis, let $y = 0$ and solve for x.
8. To find the *y-intercept*, the ordinate of the point where the graph crosses the y-axis, let $x = 0$ and solve for y.
9. The graph of the equation $y = b$ is a *horizontal line* passing through the point $(0,b)$.
10. The graph of the equation $x = a$ is a *vertical line* passing through the point $(a,0)$.
11. The **slope** m of a line is the "steepness" of the line and is defined by $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{y_1 - y_2}{x_1 - x_2}$, where $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$ are any two points lying on the line.
12. The **standard form** of the equation of a line is $ax + by = c$, where a, b, and c are integers, $a \geq 0$, and a and b not both 0.
13. The **point-slope** form of the equation of a line is given by $y - y_1 = m(x - x_1)$, where m is the slope and the point $P_1(x_1,y_1)$ lies on the line.
14. The **slope-intercept** form of the equation of a line is $y = mx + b$, where m is the slope and b is the y-intercept.

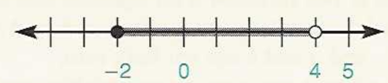
15. Two distinct nonvertical lines having slopes m_1 and m_2 are **parallel** if and only if $m_1 = m_2$. (The slopes are the same.)
16. Two nonvertical lines having slopes m_1 and m_2 are **perpendicular** if and only if $m_1 m_2 = -1$. (The slopes are negative reciprocals of each other.)
17. Vertical lines and horizontal lines are perpendicular to one another.
18. A **linear inequality in two variables** is an inequality of the form $ax + by < c$, $ax + by \leq c$, $ax + by > c$, or $ax + by \geq c$.
19. The graph of a linear inequality is a **half-plane** that
    a. includes the boundary line when the symbols $\leq$ or $\geq$ are used. The line is solid.
    b. excludes the boundary line when the symbols $<$ or $>$ are used. The line is dashed.
20. A **function** is a relation (set of ordered pairs) in which no two distinct ordered pairs have the same first component.
21. The **domain** of a function is the set of all first components of the ordered pairs in the function.
22. The **range** of a function is the set of all second components of the ordered pairs in the function.
23. Given a function f, $f(x)$ represents the value of f at the number x.

Chapter 7 error analysis

1. Ordered pair solutions of linear equations in two variables
   Example: The ordered pair (1,5) is a solution of $x = 5$.
   Correct answer: The ordered pair (5,1) is a solution of $x = 5$.
   What error was made? (*see page 278*)
2. Coordinates of points on axes
   Example: The point (0,4) lies on the x-axis.
   Correct answer: The point (4,0) lies on the x-axis.
   What error was made? (*see page 282*)
3. Ordered pairs on the graph of an equation
   Example: The ordered pair $(-1,2)$ lies on the graph of $2x - y = 3$.
   Correct answer: $(-1,2)$ is not a solution, the point does not lie on the graph.
   What error was made? (*see page 289*)
4. Finding the slope of a line
   Example: The slope of the line through $(-2,3)$ and $(2,1)$ is
   $$m = \frac{3 - 1}{2 - (-2)} = \frac{2}{2 + 2} = \frac{2}{4} = \frac{1}{2}.$$
   Correct answer: $-\frac{1}{2}$
   What error was made? (*see page 299*)
5. Zero exponents
   Example: $(a^2b^3c^0)^2 = a^4b^6c^2$
   Correct answer: a^4b^6
   What error was made? (*see page 142*)
6. Negative exponents
   Example: $(-3)^{-3} = \frac{1}{3^3} = \frac{1}{27}$
   Correct answer: $-\frac{1}{27}$
   What error was made? (*see page 141*)
7. Multiplying like bases
   Example: $3^2 \cdot 3^3 = 9^5$
   Correct answer: 3^5
   What error was made? (*see page 128*)
8. Exponents
   Exponents: $-(5)^2 = 25$
   Correct answer: -25
   What error was made? (*see page 128*)
9. Graphing linear inequalities
   Example: The graph of $-2 \leq x < 4$ is



Correct answer:



   What error was made? (*see page 118*)
10. Power to a power
   Example: $(-5x^2)^3 = -5x^5$
   Correct answer: $-125x^6$
   What error was made? (*see page 130*)

Chapter 7 critical thinking

The owner of a store has made up a work schedule for Bill and Ted so that at least one of them will be at the store each day (they never will both be off at the same time). Starting today Bill has every fourth day off and starting tomorrow Ted will have every sixth day off. Will the owner's plan work and explain why or why not.

Chapter 7 review

[7–1]

Find the value of y corresponding to the given values for x. Express the answer as an ordered pair.

1. $y = 3x + 4; x = -1, x = 0, x = 4$

2. $2x - 3y = -1; x = -2, x = 0, x = 1$

3. $y + 3 = 0; x = -7, x = 0, x = 5$

4. $5x + y = 0; x = -3, x = 0, x = 3$

Find the value of x corresponding to the given values of y. Express the answer as an ordered pair.

5. $x = -3y + 1; y = -2, y = 0, y = 5$

6. $4x + 2y = 7; y = -1, y = 0, y = 3$

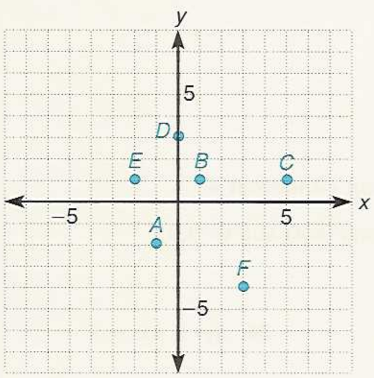
7. $x - 1 = 0; y = -8, y = 0, y = 2$

8. $3x - 2y = 0; y = -3, y = 0, y = 1$

Plot the following ordered pairs on a rectangular coordinate system.

9. $(1,5)$ **10.** $(4,-4)$ **11.** $(-1,-6)$ **12.** $(0,-4)$ **13.** $\left(2,\frac{1}{2}\right)$ **14.** $\left(5,-\frac{2}{3}\right)$

Determine to the nearest integer the coordinates, (x,y), of the given points.



15. A **16.** B **17.** C **18.** D **19.** E **20.** F

Find the missing component in each ordered pair using the given equation. Then plot the ordered pairs using a separate coordinate system for each problem.

21. $y = 3x + 4; (2,\ \ ), (0,\ \ ), \left(\frac{4}{3},\ \ \right), \left(-\frac{2}{3},\ \ \right)$

22. $y = -2x + 5; (-2,\ \ ), \left(\frac{5}{2},\ \ \right), (0,\ \ ), (3,\ \ )$

23. $y = x^2 + 2; (1,\ \ ), (-1,\ \ ), (0,\ \ ), (2,\ \ )$

24. $y = x^2 - 16; (4,\ \ ), (3,\ \ ), (0,\ \ ), (-4,\ \ )$

[7–2]

Find the x- and y-intercepts. State the answer as an ordered pair.

25. $y = 3x + 5$ **26.** $y = -4x$ **27.** $y + 2 = 0$ **28.** $x - 6 = 0$

29. $2x - 7y = 4$ **30.** $4y - x = 0$

Graph the following linear equations using the x- and y-intercepts, where possible.

31. $y = -x + 7$ **32.** $y = \frac{1}{2}x - 1$ **33.** $y = -x$ **34.** $y = 7$

35. $x = -4$ **36.** $2x - 3y = -6$ **37.** $5x - 3y = 15$

[7–3]

Find the slope of the line passing through the given pairs of points.

38. $(-4,3)$ and $(1,0)$ **39.** $(-5,3)$ and $(-5,-1)$ **40.** $(1,-2)$ and $(5,-2)$ **41.** $(-4,-4)$ and $(1,1)$

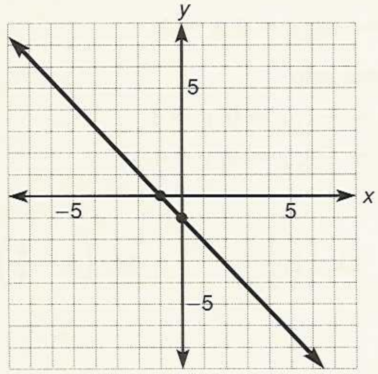
[7–4]

Express the following equations in *slope-intercept* form $y = mx + b$ and determine the slope m and y-intercept b.

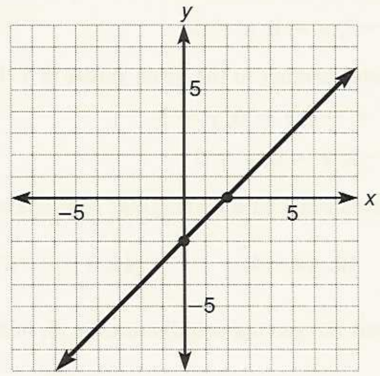
42. $3x - 4y = 8$ **43.** $4x + 3y = 2$ **44.** $8x - 3y + 1 = 0$ **45.** $y - 5x = -10$

Find the slope of the following lines and find the equation of the line using the point-slope form of the equation of a line.

46.



47.



48. A roadway rises vertically 150 feet through a horizontal distance of 1,050 feet. What is the slope of the roadway?

Find the equation of the lines having the given conditions. Write the answer in standard form.

49. Passing through (4,5) having slope $m = -4$

50. Having slope $m = \frac{1}{3}$ and y-intercept 4

51. Passing through the points (5,2) and $(-3,1)$

52. Passing through the points (0,4) and $(-3,0)$

Graph the following equations using the slope m and y-intercept b.

53. $y = 3x + 4$ **54.** $2x - y = 5$ **55.** $4y - 3x = 8$ **56.** $3y + 4x = 9$

[7–5]

Graph the following inequalities.

57. $x \leq 2y + 1$ **58.** $y > 3x - 2$ **59.** $2x - y \geq 4$ **60.** $y \leq 5$

61. $x < -1$ **62.** $2y - 5x > 0$ **63.** $3x + 2y \geq 9$ **64.** $5x - 2y < 10$

[7–6]

Determine the domain and range of the function defined by each of the following equations

65. $y = -3x - 1; x \in \{-5,-3,-1,0,1,3,5\}$

66. $x + 4y = 3; x \in \{-4,-1,0,2,3\}$

67. $x - 2y = 0; x \in \{-7,-3,0,2,6\}$

Find the indicated value of each function. State the answers as ordered pairs.

68. $f(x) = 4x; f(-1), f(0), f(4)$

69. $f(x) = 4 - x; f(-2), f(0), f(2)$

70. $f(x) = 5x - 3; f(-4), f(0), f(6)$

71. $f(x) = 5; f(-8), f(0), f(9)$

72. The perimeter P of a regular hexagon (six-sided figure whose sides are the same length and whose angles have the same measure) is given by $P = 6s$, where s is the length of a side. Express P as a function of s using f and find $f(3)$ and $f(14)$. What is the domain of f?

Chapter 7 cumulative test

[2–2] **1.** Evaluate the expression $xy + (-xy)^2 - x + y$ when $x = 3$ and $y = -2$.

[2–3] **2.** Remove the grouping symbols and simplify the expression.
$3 - [5(6x - 8) - (12x - 7) + 9x]$

Find the solution set of the following equations and inequalities.

[2–6] **3.** $6(x + 2) = 9x + 4$

[6–5] **4.** $\dfrac{3}{4} = \dfrac{5}{x} - \dfrac{2}{3}$

[2–6] **5.** $-4(y + 1) - 8 = -10 - (-3 + y)$

[6–5] **6.** $3 - \dfrac{4}{x^2} = \dfrac{4}{x}$

[2–9] **7.** $-7 \le 5 - 3x < 8$

Completely factor the following expressions.

[4–3] **8.** $7x^2 - 6x - 1$

[4–1] **9.** $y^8 + y^7 - y^6$

[4–6] **10.** $p^5 + p^4 - 4p^2 - 4p$

[4–4] **11.** $16a^2 - 16b^2$

[4–6] **12.** $t^3 - 8t^2 + 7t$

[4–1] **13.** $-35y - 28z$

Perform the indicated operations.

[3–2] **14.** $(4y - 3x)^2$

[3–2] **15.** $(4x - 7)(5x + 8)$

[3–2] **16.** $\left(5 - \dfrac{3}{4}y\right)\left(5 + \dfrac{3}{4}y\right)$

[3–2] **17.** $(x - y)^3$

[3–2] **18.** $(3y^2 + 7)(5y^3 + 6y^2 - y + 8)$

[6–2] **19.** $\dfrac{2x}{x^2 - x - 42} + \dfrac{3x}{x^2 - 49}$

[6–2] **20.** $\dfrac{a + 1}{a + 5} - \dfrac{a - 3}{a^2 + 3a - 10}$

[6–1] **21.** $\dfrac{2y - 6}{y + 8} \cdot \dfrac{3y + 24}{y^2 - 9}$

Simplify the following. Express the answer with positive exponents.

[3–3] **22.** $\dfrac{x^{-6}}{x^3}$

[3–3] **23.** $8^{-5} \cdot 8^{-3}$

[3–3] **24.** $(-5a^{-3})^3$

[3–5] **25.** Write the number 0.00000776 in scientific notation.

[5–3] **26.** Divide. $(3x^3 - 2x^2 + 4x - 5) \div (x + 2)$

[6–4] **27.** Simplify the complex fraction. $\dfrac{\dfrac{2x}{x - 3} - \dfrac{3}{x + 1}}{\dfrac{3x + 4}{x^2 - 2x - 3}}$

Graph the following equations using the x- and y-intercepts if possible.

[7–2] **28.** $2x - y = 7$

[7–2] **29.** $x - 7 = 0$

[7–5] **30.** Graph the linear inequality.
$3x - 5y > 15$

[7–3] **31.** Find the slope of the line through the points (2,3) and (−1,−4).

Find the equation of the line.

[7–4] **32.** Passing through the points (1,4) and (−3,1)

[7–4] **33.** Having slope $-\dfrac{3}{5}$ and passing through the point (4,3)

Determine if the following lines are parallel, perpendicular, or neither.

[7–4] **34.** $2x - 3y = 4$ and $4x - 6y = -1$

[7–4] **35.** $x + 2y = 6$ and $3x - 6y = 1$

[7–6] **36.** Given the function defined by $f(x) = 3x - 2$, find (a) $f(0)$, (b) $f(-3)$, (c) $f(4)$.

Solve the following word problems.

[2–8] **37.** Three-fourths of what number is 102?

[4–8] **38.** The product of a whole number times the next consecutive even whole number is 168. What is the whole number?

[4–8] **39.** A right triangle has a hypotenuse of 13 inches. If one of the legs is 7 inches longer than the other, find the lengths of the legs. (*Hint:* Use $c^2 = a^2 + b^2$, where c is the hypotenuse and a and b are the legs.)

Review exercises

1. $\{2\}$ **2.** $y = \frac{3x+6}{2}$ **3.** $8(y+2)(y-2)$

4. $(x+10)^2$ **5.** $(3y+2)(y-2)$ **6.** $\frac{5x^2+7x}{(x-1)(x+3)}$

7. $\frac{-2y^2-23y}{(2y+1)(y-5)}$

Chapter 6 review

1. $\frac{9}{2}$ **2.** $\frac{9a}{b}$ **3.** $\frac{2x-y}{2x+y}$ **4.** $x-2$ **5.** $\frac{x(m-n)}{2}$

6. $\frac{1}{y^2-2y+1}$ **7.** 2 **8.** $-\frac{1}{3x+15}$ **9.** $\frac{x-1}{x+1}$

10. $\frac{2a}{3}$ **11.** $\frac{8b}{21}$ **12.** $\frac{9}{2ab}$ **13.** $\frac{14x-7}{9x+12}$

14. $\frac{1}{x^2+4x-12}$ **15.** $\frac{10}{3}$ **16.** $\frac{1}{x-8}$ **17.** $\frac{3-b}{b^2+2b-3}$

18. $\frac{3a+3}{4a-4}$ **19.** $36x^2$ **20.** $42a^2b^2$ **21.** $2(x+3)(x-3)$

22. $(y+5)(y-5)(y+3)$ **23.** $4(z+2)(z-2)(z+1)$

24. $x(x+1)^2(3x-5)$ **25.** $\frac{3}{x}$ **26.** $\frac{4y}{y-2}$ **27.** $\frac{4x+4}{3x-1}$

28. $\frac{-x+13}{2x-5}$ **29.** $\frac{68x}{105}$ **30.** $\frac{23}{48a}$

31. $\frac{47x-6}{(3x+1)(4x-3)}$ **32.** $\frac{4x^2-23x}{(x+4)(x-4)}$

33. $\frac{80a+48b-15ab}{20a^2b^2}$ **34.** $\frac{4a^2+4a+6}{a+1}$ **35.** $\frac{-10x^2-3}{x^2+1}$

36. $\frac{16x^2+5x+15}{24x^2}$ **37.** $\frac{13y^2-89y}{(y-9)(y+2)(y-2)}$

38. $\frac{2-2ax-36a}{(x+3)(x-3)}$ **39.** $\frac{8x^2-40x-180}{x(x-5)(x+4)}$

40. $\frac{10x-5y}{(x+y)^2(x-2y)}$ **41.** $\frac{16}{63}$ **42.** $\frac{1}{7}$ **43.** 1 **44.** $\frac{y-x}{y+x}$

45. $\frac{4+3x}{2-5x}$ **46.** $\frac{a^2-b^2}{a^2+b^2}$ **47.** $y-x$ **48.** $\frac{xy}{x+y}$

49. $\frac{a^3-2a^2b^2-3ab^2}{ab-3b^2}$ **50.** $\{-96\}$ **51.** $\left\{\frac{41}{48}\right\}$ **52.** $\left\{\frac{13}{24}\right\}$

53. $\left\{\frac{6}{5}\right\}$ **54.** $\left\{-\frac{39}{14}\right\}$ **55.** $\left\{\frac{7}{52}\right\}$ **56.** $\{0,12\}$

57. $\left\{-\frac{3}{2},\frac{3}{2}\right\}$ **58.** $\{-5,3\}$ **59.** $\left\{6,\frac{3}{2}\right\}$ **60.** $x = \frac{a+b}{3}$

61. $x = \frac{y+3}{1-y}$ **62.** $y = \frac{4a-3b}{a}$ **63.** $y = 4b+5c$

64. $l = \frac{Aey}{F}$ **65.** $L = \frac{Wp-2E}{EF}$ **66.** $R = \frac{Mr}{M-2}$

67. $m = \frac{Fr}{v^2+gr}$ **68.** $5\frac{1}{3}$ hr, 16 hr **69.** $1\frac{5}{7}$ mph **70.** $\frac{2}{3}$ or 1

Chapter 6 cumulative test

1. $-\frac{71}{12}$ **2.** $\frac{3}{4}$ **3.** $\frac{43}{30}$ **4.** $\frac{8}{5}$ **5.** x^{11} **6.** $\frac{1}{y^9}$ **7.** $\frac{25x^4}{y^6}$

8. $10x^3-4x^2+11x-9$ **9.** $\left\{-\frac{26}{7}\right\}$ **10.** $\left\{\frac{81}{4}\right\}$

11. $\left\{-\frac{3}{2},2\right\}$ **12.** $x \geq -5$ **13.** $2(x+3)(x-3)$

14. $3x^2(2x^3-12x+3)$ **15.** $(2x+5)(2x+3)$

16. $3(1+2x^3)(1-2x^3)$ **17.** $49x^2-84x+36$

18. $25-\frac{1}{4}x^2$ **19.** $6x^3-19x^2+37x-33$

20. $15-22x^2+8x^4$ **21.** $x = \frac{120}{7}$

22. 2,400 foot-pounds/min **23.** $\frac{4a^2cf\ell}{5b^2}$ **24.** $\frac{y^2+10y+21}{y^2-8y+12}$

25. y^2-5y+6 **26.** $\frac{(x-y)(x-1)}{y}$ **27.** $\frac{2y+4x}{x^2y^2}$

28. $\frac{3x+9y}{(x-y)(x+y)}$ **29.** $\frac{9}{a-2}$ **30.** $\frac{13x+56}{(x-7)(x+7)(x+2)}$

31. $\frac{-x^2-6x}{(x+3)(x-2)(x-6)}$ **32.** $\frac{3y-1}{4y+5}$ **33.** $\frac{y+x}{3x-4y}$

34. $\frac{x^2-11x+28}{x-5}$ **35.** $\{2\}$ **36.** $\left\{-\frac{1}{3},6\right\}$ **37.** $q = \frac{fp}{p-f}$

38. $2\frac{2}{5}$ days **39.** $\frac{8}{16}$

Chapter 7

Exercise 7-1

Answers to odd-numbered problems

1. (1,2) and (−1,−4) are solutions. **3.** (−1,2), (3,0) are solutions. **5.** (1,2) is a solution. **7.** (2,3), (0,0) are solutions. **9.** (−4,1) and (−4,−4) are solutions. **11.** (−5,3), (−5,8) are solutions. **13.** (1,5), (−2,−4), (0,2) **15.** (3,−3), (−4,11), (0,3)

17. (3,−5), (−2,10), (0,4) **19.** (−2,−1), (3,0), $\left(0,-\frac{3}{5}\right)$

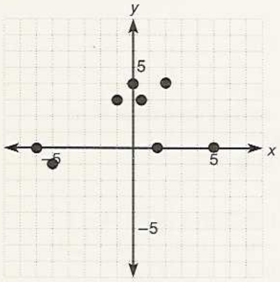
21. (1,−4), (−1,1), $\left(0,-\frac{3}{2}\right)$ **23.** (1,5), (−6,5), (0,5)

25. (7,−1), $\left(-\frac{3}{5},-1\right)$, (0,−1) **27.** (5,1), (−1,−2), (3,0)

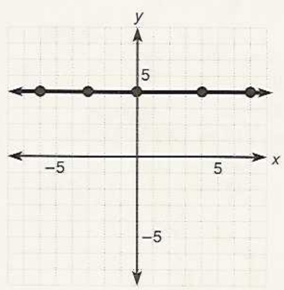
29. (3,2), (−6,−4), (0,0) **31.** (1,−2), (1,7), (1,0)
33. a. (75,\$170) **b.** (300,\$620) **c.** (1,000,\$2,020) **35. a.** (2,236) **b.** (12,216) **c.** (0,240) **37. a.** (0,20) **b.** (5,35) **c.** (3,29)

39. a. (3,165) **b.** (8,440) **c.** $\left(\frac{26}{5},286\right)$

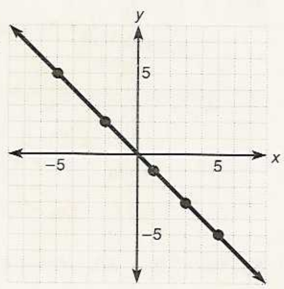
41. (2,4) **43.** $(-1,3)$ **45.** $(-6,-1)$ **47.** (0,4) **49.** (5,0)
51. $(-7,0)$ **53.** $\left(\frac{1}{2},3\right)$ **55.** $\left(\frac{3}{2},0\right)$



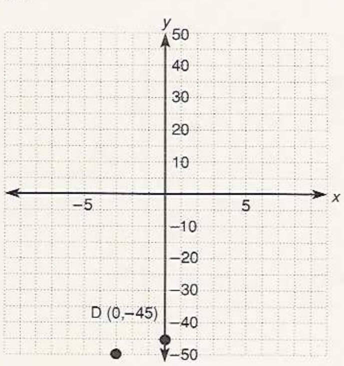
57. $B(2,-1)$ **59.** $D(0,6)$ **61.** $F(0,-6)$ **63.** $H(-4,4)$
65. $J(5,-5)$ **67.** III **69.** I **71.** IV **73.** III **75.** IV
77. a. IV **b.** II **c.** III **d.** I **79.** 0 **81.** B, -1; D, 6; F, -6; H, 4; J, -5 **83.** Figure is a horizontal line parallel to the x-axis having equation $y = 4$.



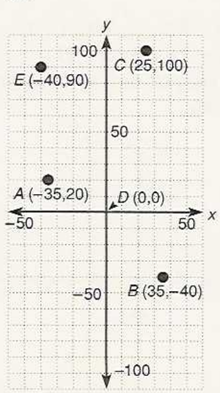
85. $(-5,5)$, $(3,-3)$, $(-2,2)$, $(5,-5)$, $(1,-1)$ The resulting figure is a line through the origin that lies in quadrants II and IV.



87.

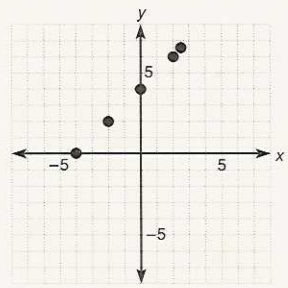


89.

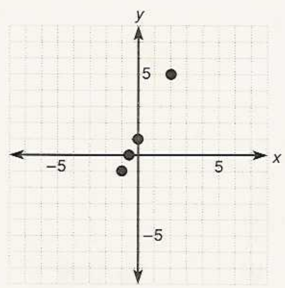


91. A (Tues., 70), the temperature was 70° on Tuesday. B (Wed., 80), the temperature was 80° on Wednesday. C (Fri., 90), the temperature was 90° on Friday. D (Sat., 85), the temperature was 85° on Saturday.

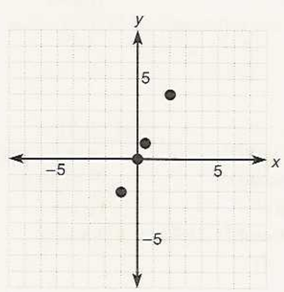
93. (0,4), $(-4,0)$, $\left(\frac{5}{2},\frac{13}{2}\right)$, (2,6)



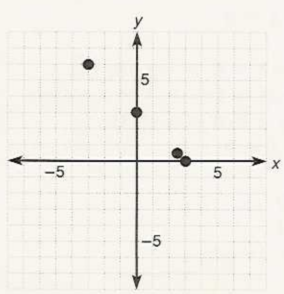
95. (0,1), $\left(-\frac{1}{2},0\right)$, (2,5), $(-1,-1)$



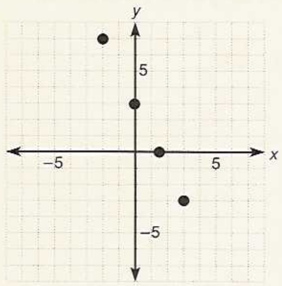
97. (0,0), $\left(\frac{1}{2},1\right)$, (2,4), $(-1,-2)$



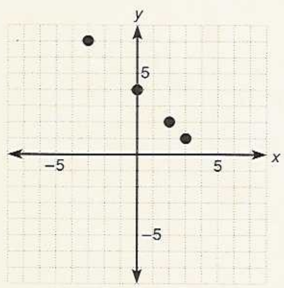
99. (0,3), $(-3,6)$, (3,0), $\left(\frac{5}{2},\frac{1}{2}\right)$



101. (0,3), (3,−3), (−2,7), $\left(\frac{3}{2},0\right)$



103. (0,4), (−3,7), (2,2), (3,1)



Solutions to trial exercise problems

1. $y = 3x - 1$; (1,2), (−1,−4), (2,3)

Solution:

(a) (1,2)	(b) (−1,−4)	(c) (2,3)
$2 = 3(1) - 1$	$-4 = 3(-1) - 1$	$3 = 3(2) - 1$
$2 = 3 - 1$	$-4 = -3 - 1$	$3 = 6 - 1$
$2 = 2$	$-4 = -4$	$3 = 5$
(true)	(true)	(false)

Therefore (1,2) and (−1,−4) are solutions and (2,3) is not a solution.

7. $3x = 2y$; (2,3), (3,2), (0,0)

Solution:

(a) (2,3)	(b) (3,2)	(c) (0,0)
$3(2) = 2(3)$	$3(3) = 2(2)$	$3(0) = 2(0)$
$6 = 6$	$9 = 4$	$0 = 0$
(true)	(false)	(true)

So (2,3) and (0,0) are solutions and (3,2) is not a solution.

9. $x = -4$; (−4,1), (4,2), (−4,−4)

Solution:

Since we can write $x = -4$ as $x = 0 \cdot y - 4$, then $x = -4$ for *any* value of y. Therefore (−4,1) and (−4,−4) are solutions but (4,2) is not since, substituting 4 for x, $4 = -4$ is false.

15. $y = 3 - 2x$; $x = 3$, $x = -4$, $x = 0$

Solution:

When $x = 3$, $y = 3 - 2(3) = 3 - 6 = -3$ (3,−3)

When $x = -4$, $y = 3 - 2(-4) = 3 + 8 = 11$ (−4,11)

When $x = 0$, $y = 3 - 2(0) = 3 - 0 = 3$ (0,3)

20. $x + 4y = 0$; $x = -4$, $x = 8$, $x = 0$

Solution:

When $x = -4$, $-4 + 4y = 0$
$$4y = 4$$
$$y = 1 \quad (-4,1)$$

When $x = 8$, $8 + 4y = 0$
$$4y = -8$$
$$y = -2 \quad (8,-2)$$

When $x = 0$, $0 + 4y = 0$
$$4y = 0$$
$$y = 0 \quad (0,0)$$

23. $y = 5$; $x = 1$, $x = -6$, $x = 0$

Solution:

We write $y = 5$ as $y = 0 \cdot x + 5$, so for *every* value of x, $y = 5$. Therefore (1,5), (−6,5), (0,5).

28. $x = -3y + 1$; $y = -1$, $y = 2$, $y = 0$

Solution:

When $y = -1$, $x = -3(-1) + 1 = 3 + 1 = 4$ (4,−1)

When $y = 2$, $x = -3(2) + 1 = -6 + 1 = -5$ (−5,2)

When $y = 0$, $x = -3(0) + 1 = 0 + 1 = 1$ (1,0)

31. $x = 1$; $y = -2$, $y = 7$, $y = 0$

Solution:

Since $x = 1$ can be written $x = 0 \cdot y + 1$, then $x = 1$ for *any* value of y, so we have (1,−2), (1,7), (1,0).

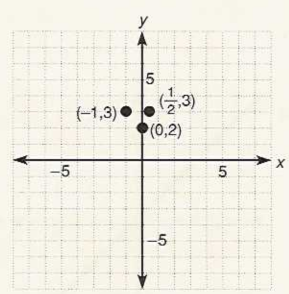
36. Given $y = 240 - 2x$, when

a. $y = 200$, then $200 = 240 - 2x$
$$-40 = -2x$$
$$20 = x \qquad (20,200)$$

b. $y = 0$, then $0 = 240 - 2x$
$$-240 = -2x$$
$$120 = x \qquad (120,0)$$

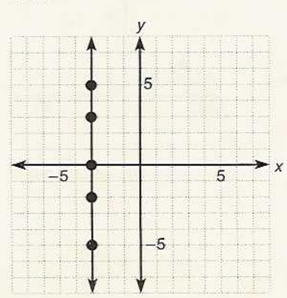
c. $y = 210$, then $210 = 240 - 2x$
$$-30 = -2x$$
$$15 = x \qquad (15,210)$$

43. **48.** **53.**

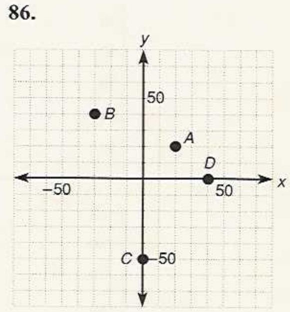


58. $C(-2,0)$ **63.** $H(-4,4)$ **64.** $I(7,0)$ **68.** (4,−1) lies in quadrant IV. **73.** $\left(-\frac{5}{2}, -\frac{3}{4}\right)$ lies in quadrant III.

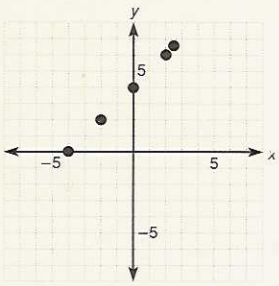
82. (−3,3), (−3,0), (−3,−2), (−3,5), (−3,−5)
They lie on a vertical straight line.



86.



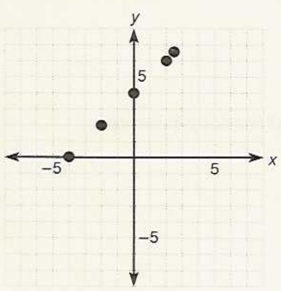
91. A (Tues., 70), the temperature was 70° on Tuesday.
B (Wed., 80), the temperature was 80° on Wednesday.
C (Fri., 90), the temperature was 90° on Friday.
D (Sat., 85), the temperature was 85° on Saturday.



93. When $x = 0$, $y = 0 + 4 = 4$ $(0,4)$
When $x = -4$, $y = -4 + 4 = 0$ $(-4,0)$

When $x = \frac{5}{2}$, $y = \frac{5}{2} + 4 = \frac{13}{2}$ $\left(\frac{5}{2},\frac{13}{2}\right)$

When $x = 2$, $y = 2 + 4 = 6$ $(2,6)$



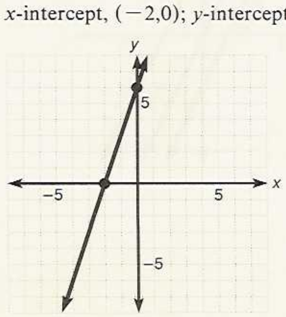
Review exercises

1. $2x^2 + 4x$ **2.** $\dfrac{x+3}{2x+1}$ **3.** $\dfrac{x+10}{(x-2)(x+1)}$ **4.** -8 **5.** x^3
6. 0

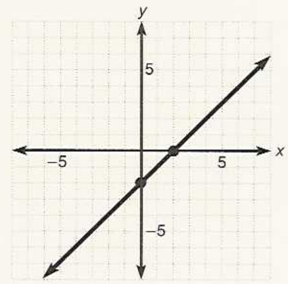
Exercise 7–2

Answers to odd-numbered problems

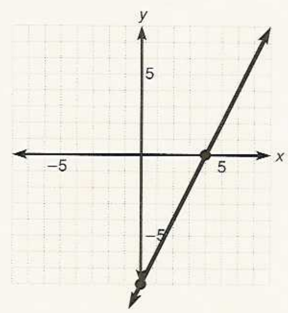
1. x-intercept, $(-2,0)$; y-intercept, $(0,4)$ **3.** x-intercept, $\left(-\frac{1}{3},0\right)$; y-intercept, $(0,1)$ **5.** x-intercept, $(3,0)$; y-intercept, $(0,2)$ **7.** x-intercept, $\left(\frac{11}{2},0\right)$; y-intercept, $\left(0,\frac{11}{5}\right)$ **9.** x-intercept, $(0,0)$; y-intercept, $(0,0)$ **11.** x-intercept, $(0,0)$; y-intercept, $(0,0)$
13. x-intercept, $(4,0)$; y-intercept, $(0,2)$ **15.** x-intercept, $(3,0)$; y-intercept, $\left(0,-\frac{3}{2}\right)$ **17.** x-intercept, $(-2,0)$; y-intercept, $(0,6)$



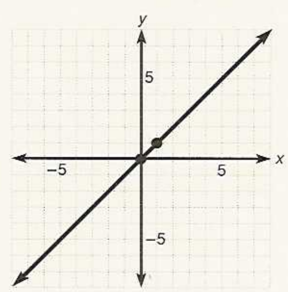
19. x-intercept, $(2,0)$; y-intercept, $(0,-2)$



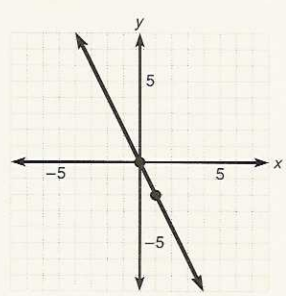
21. x-intercept, $(4,0)$; y-intercept, $(0,-8)$



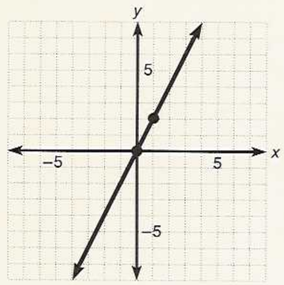
23. x-intercept, $(0,0)$; y-intercept, $(0,0)$; $(1,1)$



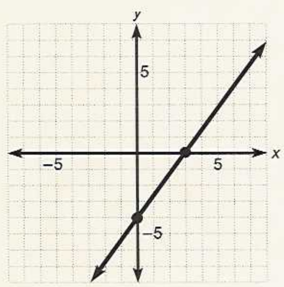
25. x-intercept, $(0,0)$; y-intercept, $(0,0)$; $(1,-2)$



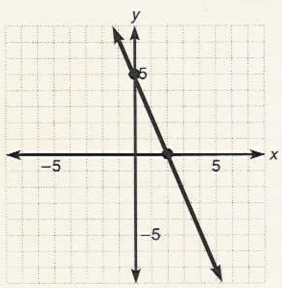
27. x-intercept, (0,0); y-intercept, (0,0); (1,2)



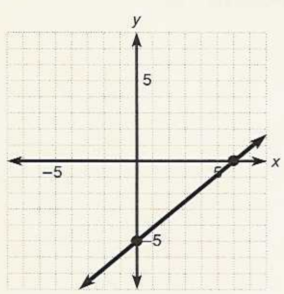
29. x-intercept, (3,0); y-intercept, (0,−4)



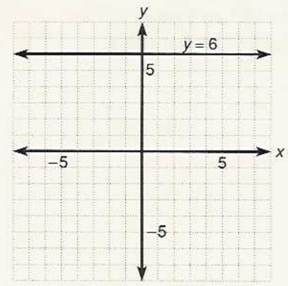
31. x-intercept, (2,0); y-intercept, (0,5)



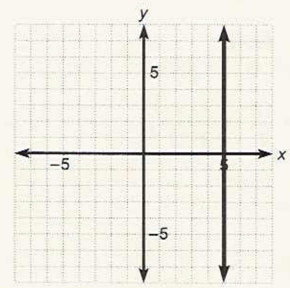
33. x-intercept, (6,0); y-intercept, (0,−5)



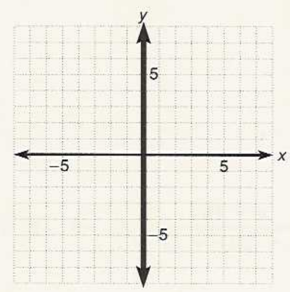
35. no x-intercept; y-intercept, (0,6)



37. x-intercept, (5,0); no y-intercept



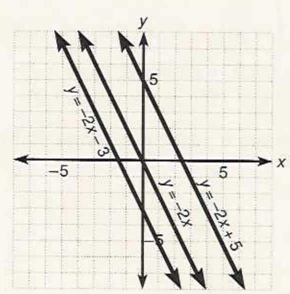
39. x-intercept, (0,0); All real numbers are y-intercepts.



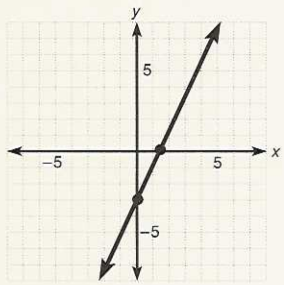
41. $y = 2x - 7$ **43.** $y = \frac{4}{3}x + 3$ **45.** $y = -\frac{7}{3}x + \frac{10}{3}$

47. $y = \frac{1}{5}x + \frac{7}{5}$ **49.** $y = \frac{8}{5}x - \frac{14}{5}$

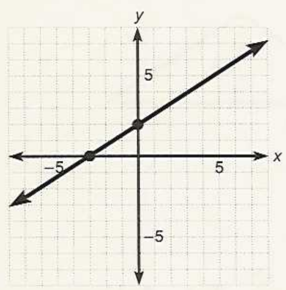
51. a. $y = -2x + 5$ **b.** $y = -2x$ **c.** $y = -2x - 3$



53. $y = 2x - 3$; x-intercept $\left(\frac{3}{2},0\right)$; y-intercept, $(0,-3)$



55. $3y - 2x = 6$; x-intercept, $(-3,0)$;
y-intercept, $(0,2)$



Solutions to trial exercise problems

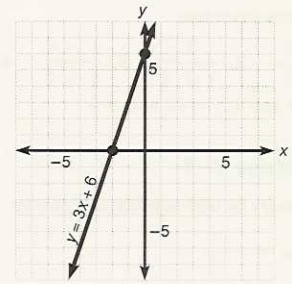
1. $y = 2x + 4$
Let $x = 0$,
then $y = 2(0) + 4 = 0 + 4 = 4$ $\quad$ (0,4) is the y-intercept.
Let $y = 0$,
then $0 = 2x + 4$, $2x = -4$
$x = -2$ $\quad$ $(-2,0)$ is the x-intercept.

5. $2x + 3y = 6$
Let $x = 0$, then
$2(0) + 3y = 6$
$0 + 3y = 6$
$3y = 6$
$y = 2$ $\quad$ (0,2) is the y-intercept.
Let $y = 0$, then
$2x + 3(0) = 6$
$2x + 0 = 6$
$2x = 6$
$x = 3$ $\quad$ (3,0) is the x-intercept.

12. $y - 4x = 0$
Let $x = 0$, then
$y - 4(0) = 0$
$y - 0 = 0$
$y = 0$ $\quad$ (0,0) is the y-intercept.
Let $y = 0$, then
$0 - 4x = 0$
$-4x = 0$
$x = 0$ $\quad$ (0,0) is the x-intercept.

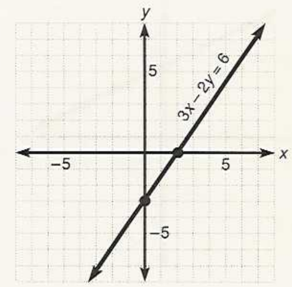
17. $y = 3x + 6$

x	y	
0	6	y-intercept
-2	0	x-intercept
-1	3	checkpoint



30. $3x - 2y = 6$

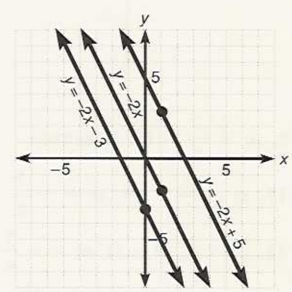
x	y	
0	-3	y-intercept
2	0	x-intercept
1	$-\frac{3}{2}$	checkpoint



41. $y - 2x + 7 = 0$
$y - 2x = -7$ $\quad$ Add -7 to both members.
$y = 2x - 7$ $\quad$ Add $2x$ to both members.

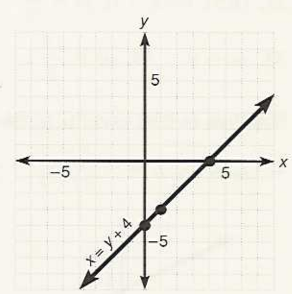
43. $3y - 4x = 9$
$3y = 4x + 9$ $\quad$ Add $4x$ to both members.
$y = \dfrac{4x + 9}{3}$ $\quad$ Divide both members by 3.
$y = \dfrac{4}{3}x + 3$ $\quad$ Divide by 3.

51. When $b = 5$, $y = -2x + 5$
When $b = 0$, $y = -2x$
When $b = -3$, $y = -2x - 3$



54. $x = y + 4$

x	y	
0	-4	y-intercept
4	0	x-intercept
1	-3	checkpoint



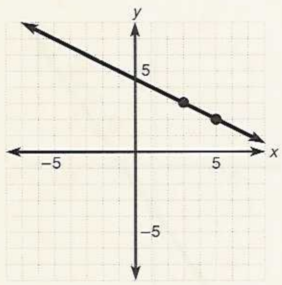
Review exercises

1. $\frac{6}{5}$ **2.** $\frac{1}{x+4}$ **3.** $\frac{x-4}{x+5}$ **4.** $-\frac{1}{5}$ **5.** $y = -3x + 6$
6. 4 and 12; -12 and -4

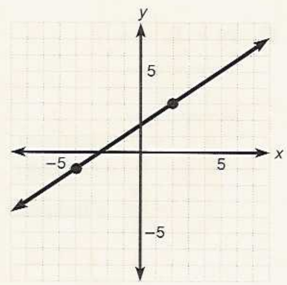
Exercise 7–3

Answers to odd-numbered problems

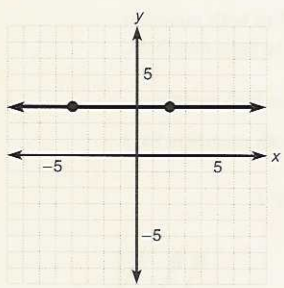
1. $m = -\frac{1}{2}$



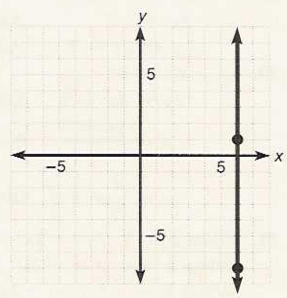
3. $m = \frac{2}{3}$



5. $m = 0$



7. m is undefined.



9. $m = \frac{2}{3}$ **11.** undefined **13.** $m = 1$ **15.** undefined

17. $m = \frac{1}{7}$ **19.** $m = -\frac{3}{4}$ **21.** $m = \frac{10}{11}$

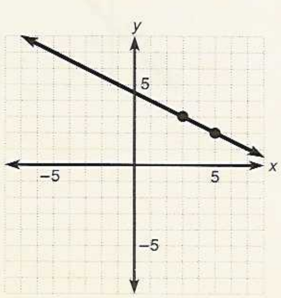
23. $(-5,3)$, $(4,-2)$; $m = -\frac{5}{9}$ **25.** $(3,5)$, $(-2,-6)$; $m = \frac{11}{5}$

27. $(6,5)$, $(-3,-5)$; $m = \frac{10}{9}$ **29.** $m = \frac{2}{3}$ **31.** $m = \frac{2}{3}$

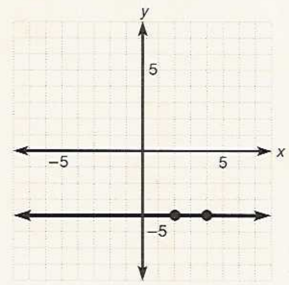
33. $m = 4$ **35.** $m = \frac{15}{2}$

Solutions to trial exercise problems

1. $(5,2)$, $(3,3)$; $m = \frac{3-2}{3-5} = \frac{1}{-2} = -\frac{1}{2}$



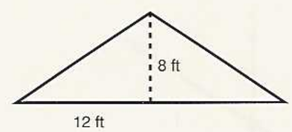
6. $(4,-4)$, $(2,-4)$; $m = \frac{-4-(-4)}{4-2} = \frac{0}{2} = 0$



15. $(0,7)$, $(0,-8)$; $m = \frac{7-(-8)}{0-0} = \frac{15}{0}$ slope is undefined

23. Using points $(-5,3)$ and $(4,-2)$, $m = \frac{3-(-2)}{-5-4} = \frac{5}{-9} = -\frac{5}{9}$

29. $m = \frac{8}{12} = \frac{2}{3}$



33. $m = \frac{1{,}000}{250} = \frac{4}{1} = 4$

Review exercises

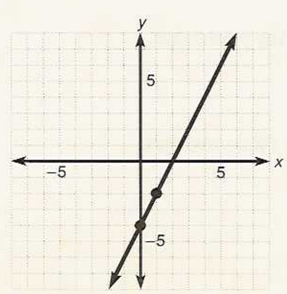
1. -9 **2.** -13 **3.** $-\frac{1}{8}$ **4.** $\frac{b^5}{a^5}$ **5.** x **6.** 13
7. $y = -3x - 1$ **8.** $y = x - 2$

Exercise 7–4

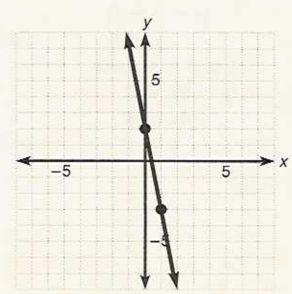
Answers to odd-numbered problems

1. $y = 4x - 1$ **3.** $y = -2x - 1$ **5.** $5y - 3x = 29$
7. $6y + 7x = 35$ **9.** $4y - 5x = -13$ **11.** $y = -3$
13. $x = 1$ **15.** $x - 2y = 0$ **17.** $3x + 8y = 7$
19. $-x + y = 4$ **21.** $x + 7y = -6$ **23.** $-8x + 3y = 24$
25. $8x + 5y = 0$ **27.** $y = -x + 2$; $m = -1$; y-intercept, 2
29. $y = -3x - 2$; $m = -3$; y-intercept, -2
31. $y = -\frac{2}{5}x + 2$; $m = -\frac{2}{5}$; y-intercept, 2
33. $y = \frac{7}{2}x + 2$; $m = \frac{7}{2}$; y-intercept, 2 **35.** $y = \frac{9}{2}x - 3$; $m = \frac{9}{2}$; y-intercept, -3 **37.** $y = \frac{8}{9}x - \frac{1}{9}$; $m = \frac{8}{9}$; y-intercept, $-\frac{1}{9}$

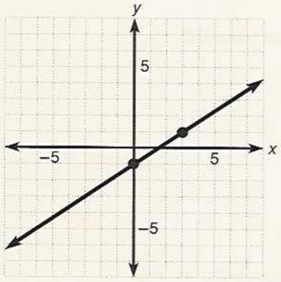
39. $m = 2$; $b = -4$



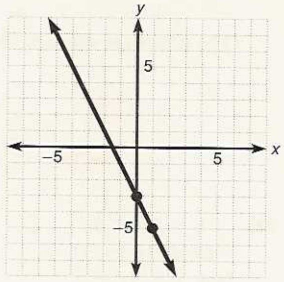
41. $m = -5$; $b = 2$



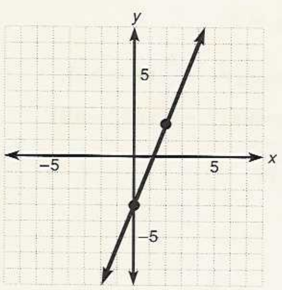
43. $m = \frac{2}{3}$; $b = -1$



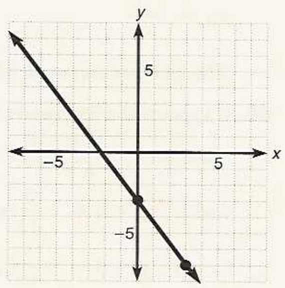
45. $m = -2$; $b = -3$



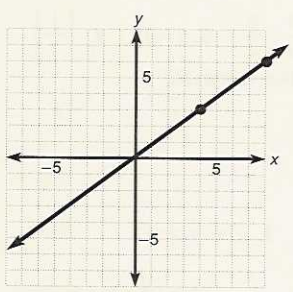
47. $m = \frac{5}{2}$; $b = -3$



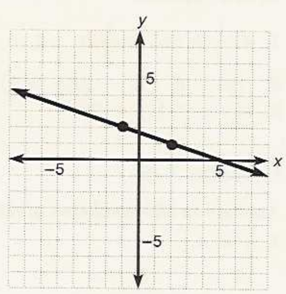
49. $m = -\frac{4}{3}$; $b = -3$



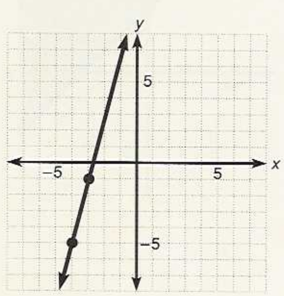
51. $(4,3)$; $m = \frac{3}{4}$



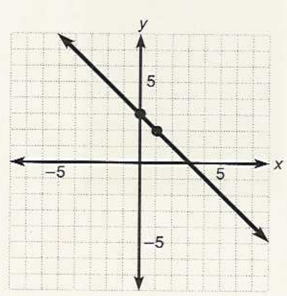
53. $(-1,2)$; $m = -\frac{1}{3}$



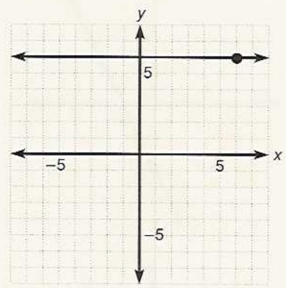
55. $(-4,-5)$; $m = 4$



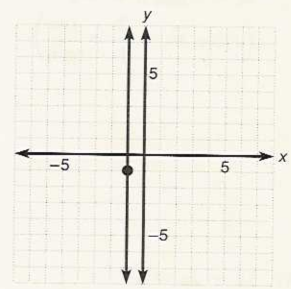
57. $(0,3)$; $m = -1$



59. $(5,6)$; $m = 0$



61. $(-1,-1)$; undefined slope



63. $y = 4x - 1$ **65.** $y = -\frac{5}{3}x + 2$ **67.** $y = -1$

69. $m_1 = -1$, $m_2 = -1$; parallel **71.** $m_1 = -1$, $m_2 = 1$; perpendicular **73.** $m_1 = 3$, $m_2 = -3$; neither
75. $m_1 = -5$, $m_2 = -\frac{10}{3}$; neither **77.** $m_1 = 4$, $m_2 = 4$; parallel
79. $m_1 = 0$, $m_2 =$ undefined; perpendicular **81.** $(-6,2)$, $(7,-3)$; $5x + 13y = -4$ **83.** $(3,-2)$, $(-4,-2)$; $y = -2$
85. $(1,-4)$, $(-6,4)$; $8x + 7y = -20$

Solutions to trial exercise problems

1. $(1,3)$; $m = 4$

Using $y - y_1 = m(x - x_1)$

$$y - 3 = 4(x - 1)$$
$$y - 3 = 4x - 4$$
$$y = 4x - 1$$

8. $(-3,0)$; $m = -\frac{5}{8}$

$$y - 0 = -\frac{5}{8}[x - (-3)]$$
$$8y = -5(x + 3)$$
$$8y = -5x - 15$$
$$8y + 5x = -15$$

15. $(2,1)$ and $(6,3)$

$$m = \frac{3 - 1}{6 - 2} = \frac{2}{4} = \frac{1}{2}$$

Using $y - y_1 = m(x - x_1)$ and point $(2,1)$

$$y - 1 = \frac{1}{2}(x - 2)$$
$$2y - 2 = 1(x - 2)$$
$$2y - x = 0$$

23. $(0,8)$ and $(-3,0)$

$$m = \frac{8 - 0}{0 - (-3)} = \frac{8}{3}$$

Using $y - y_1 = m(x - x_1)$ and point $(0,8)$

$$y - 8 = \frac{8}{3}(x - 0)$$
$$3y - 24 = 8x$$
$$3y - 8x = 24$$

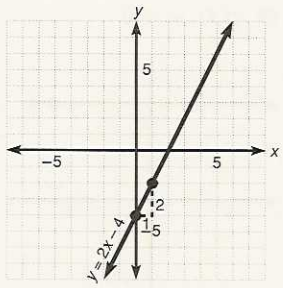
29. $3x + y = -2$ $\quad m = -3$

$y = -3x - 2$ $\quad b = -2$

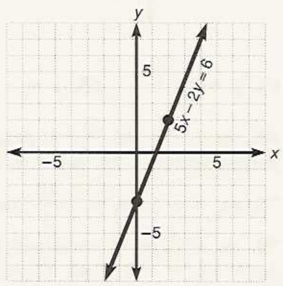
31. $2x + 5y = 10$

$$5y = -2x + 10$$
$$y = \frac{-2x + 10}{5} \qquad m = -\frac{2}{5}$$
$$y = -\frac{2}{5}x + 2 \qquad b = 2$$

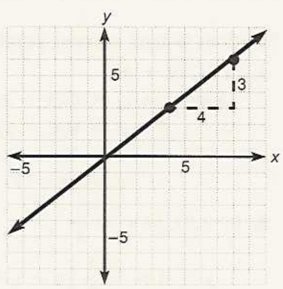
39. $y = 2x - 4$;
$m = 2$ and $b = -4$
$\left(\textit{Note: } m = \dfrac{2}{1} = \dfrac{\text{rise}}{\text{run}}\right)$



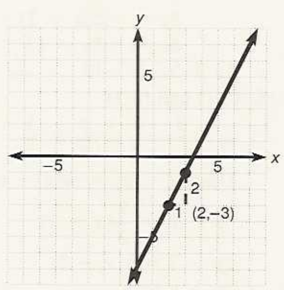
47. $5x - 2y = 6$
$-2y = -5x + 6$
$y = \dfrac{-5x + 6}{-2}$
$y = \dfrac{5}{2}x - 3$
$m = \dfrac{5}{2}$ and $b = -3$



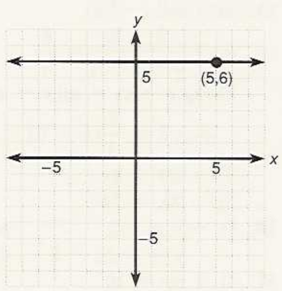
51. $(4,3)$; $m = \dfrac{3}{4}$



56. $(2,-3)$; $m = 2 = \dfrac{2}{1}$



59. $(5,6)$; $m = 0$



69. $x + y = 4$
$x + y = -7$
Then $y = -x + 4$ $\quad m = -1$
$y = -x - 7$ $\quad m = -1$
The slopes are equal so the lines are *parallel.*

74. $x + 2y = 5$
$-6x + 3y = -1$
Now $x + 2y = 5$ $\qquad -6x + 3y = -1$
$2y = -x + 5$ $\qquad 3y = 6x - 1$
$y = -\dfrac{1}{2}x + \dfrac{5}{2}$ $\qquad y = 2x - \dfrac{1}{3}$
$m_1 = -\dfrac{1}{2}$ $\qquad m_2 = 2$

The lines are *perpendicular* since $m_1m_2 = -\dfrac{1}{2} \cdot 2 = -1$.

81. It appears the line passes through points $(7,-3)$ and $(-6,2)$.
Then $m = \dfrac{2 - (-3)}{-6 - 7} = \dfrac{5}{-13} = \dfrac{-5}{13}$
and $y - (-3) = -\dfrac{5}{13}(x - 7)$
$y + 3 = -\dfrac{5}{13}(x - 7)$
$13y + 39 = -5x + 35$
$5x + 13y = -4$

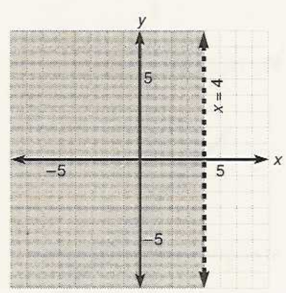
Review exercises

1. $41\dfrac{1}{2}$ ft and $58\dfrac{1}{2}$ ft **2.** $0, -3, 9, \dfrac{9}{3}, \dfrac{0}{4}$ **3.** $\{-3\}$ **4.** $\left\{\dfrac{1}{7}\right\}$
5. $\{10\}$ **6.** $(3x - 2)(x + 2)$ **7.** Not factorable
8. $8(y + 2x)(y - 2x)$

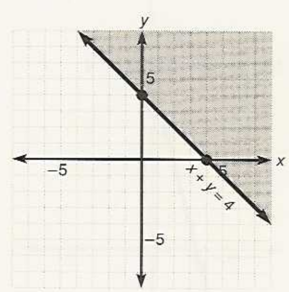
Exercise 7–5

Answers to odd-numbered problems

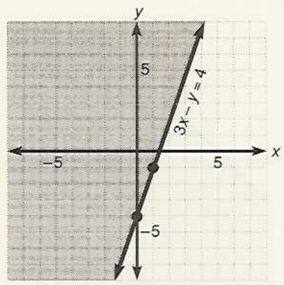
1. $x < 4$



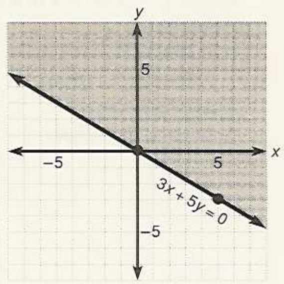
3. $x + y \geq 4$



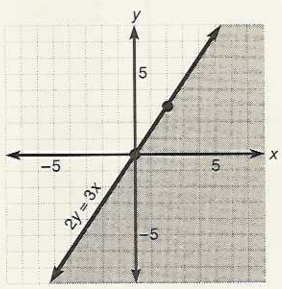
5. $3x - y \le 4$



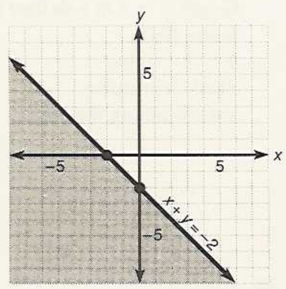
7. $3x + 5y \ge 0$



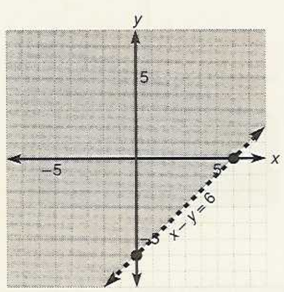
9. $2y \le 3x$



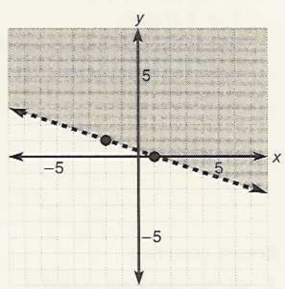
11. $x + y \le -2$



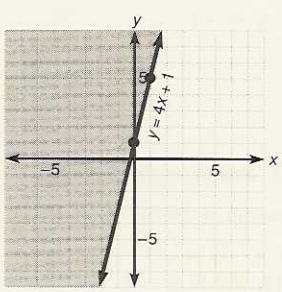
13. $x - y < 6$



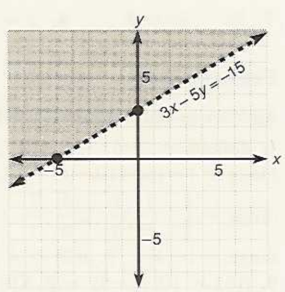
15. $x + 3y > 1$



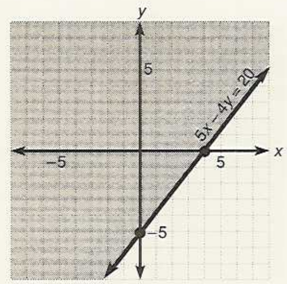
17. $y \ge 4x + 1$



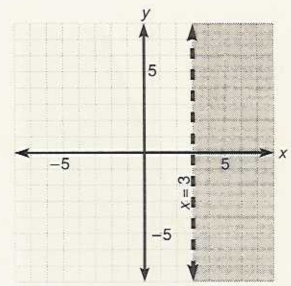
19. $3x - 5y < -15$



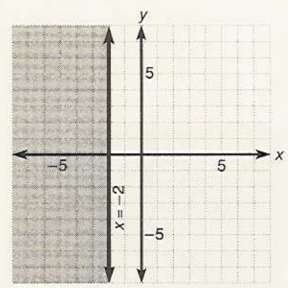
21. $5x - 4y \le 20$



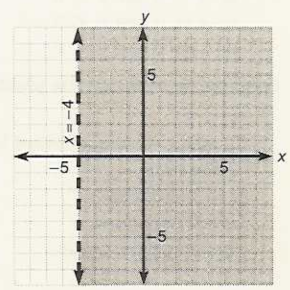
23. $x > 3$



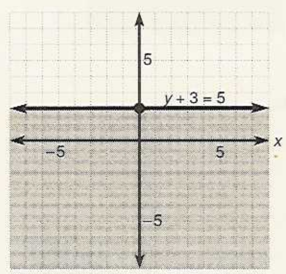
25. $x \le -2$



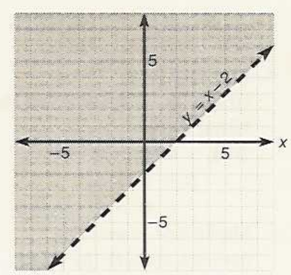
27. $x + 4 > 0$



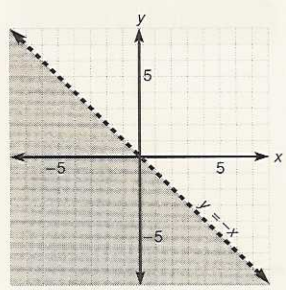
29. $y + 3 \le 5$



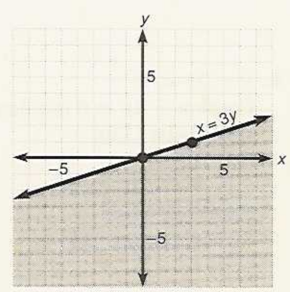
31. $y > x - 2$



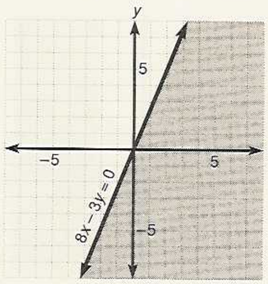
33. $x < -y$



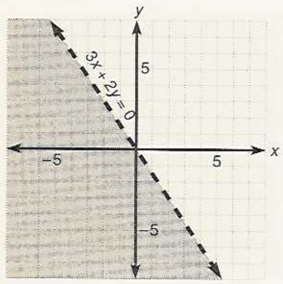
35. $x \ge 3y$



37. $8x - 3y \geq 0$



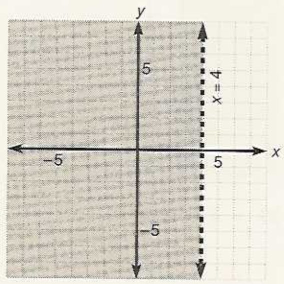
39. $3x + 2y < 0$



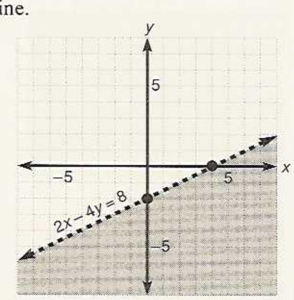
41. **a.** $x > 0$ and $y > 0$
b. $x < 0$ and $y > 0$
c. $x < 0$ and $y < 0$
d. $x > 0$ and $y < 0$

Solutions to trial exercise problems

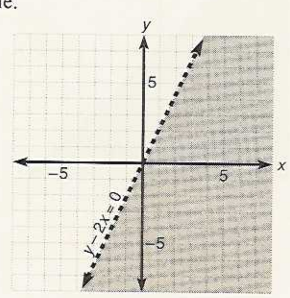
1. $x < 4$
Graph $x = 4$ in a dashed line.
Using (0,0),
$0 < 4$ (true)
Shade half-plane containing the origin.



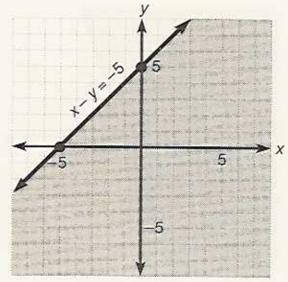
6. $2x - 4y > 8$
Graph $2x - 4y = 8$ in a dashed line.
Using (0,0),
$2(0) - 4(0) > 8$
$0 - 0 > 8$
$0 > 8$ (false)
Shade half-plane that does not contain the origin.



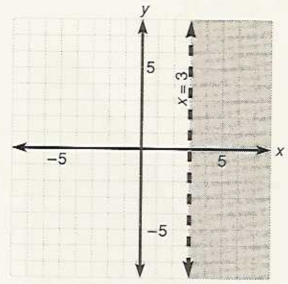
8. $y - 2x < 0$
Graph $y - 2x = 0$ in a dashed line.
Using point (3,0),
$0 - 2(3) < 0$
$-6 < 0$ (true)
Shade half-plane containing point (3,0).



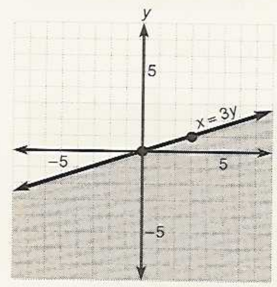
14. $x - y \geq -5$
Graph $x - y = -5$ in a *solid* line.
Using (0,0),
$0 - 0 \geq -5$
$0 \geq -5$ (true)
Shade half-plane containing the origin.



23. $x > 3$
Graph $x = 3$ in a dashed line.
Using (0,0),
$0 > 3$ (false)
Shade half-plane *not* containing the origin.



35. $x \geq 3y$
Graph $x = 3y$ in a solid line.
Using point (1,1),
$1 \geq 3$ (false)
Shade the half-plane that does not contain the point (1,1).



Review exercises

1. all real numbers except 3 **2.** all real numbers except -5 and 2 **3.** all real numbers **4.** $y = \frac{5}{2}$ **5.** $x = 13$ **6.** $x + 3y = 7$ **7.** $3x - 4y = 8$ **8.** $x = -3$ **9.** $y = -3$

Exercise 7–6

Answers to odd-numbered problems

1. domain $= \{-5,-3,-1,0,1,3,5\}$, range $= \{-16,-10,-4,-1,2,8,14\}$ **3.** domain $= \{0,1,2,3,4,5\}$, range $= \left\{-\frac{1}{3},\frac{1}{3},1,\frac{5}{3},\frac{7}{3},3\right\}$ **5.** domain $= \{-10,-5,0,5,10\}$, range $= \{-4\}$ **7.** $f(2) = 4, f(0) = 0, f(-1) = -2$ **9.** $f(5) = 10, f(-5) = 0, f(0) = 5$ **11.** $f(-3) = -4$, $f\left(-\frac{5}{3}\right) = 0, f(1) = 8$ **13.** $f(-7) = 3, f(0) = 3, f(0.41) = 3$ **15.** $(-3,-14), \left(-\frac{2}{5},-1\right), (0,1), (4,21), \left(\frac{1}{5},2\right)$ **17.** $(6,5), (-3,-1), (0,1), \left(\frac{3}{2},2\right), \left(-\frac{3}{2},0\right)$

19. $(-1,0)$, $(-10,0)$, $(0,0)$, $\left(\frac{8}{3},0\right)$, $(2,0)$ **21.** $f(100) = 212$, $f(0) = 32$, $f(25) = 77$, $f(-10) = 14$ **23.** $h(2) = 58$, $h(3) = 87$, $h(4) = 116$, domain $= \{$positive real numbers$\}$
25. $g(2) = 0.14$, $g(3) = 0.21$, $g(1.2) = 0.084$
27. $g(N) = 0.7N$, $g(5) = 3.5$, $g(10) = 7$, $g(3) = 2.1$
29. $h(n) = \frac{n}{4} + 40$, $h(50) = 52.5$, $h(12) = 43$, $h(120) = 70$
31. $g(h) = 10.5\ h$, $g(40) = 420$, $g(48) = 504$, $g(28) = 294$
33. $f(C,M) = C + M$; $f(25,7) = 32$, $f(32,11) = 43$, $f(146,27) = 173$

Solutions to trial exercise problems

3. $2x - 3y = 1$; $x \in \{0,1,2,3,4,5\}$
The domain is $\{0,1,2,3,4,5\}$ and since when we solve for y we get $y = \frac{2x-1}{3}$, then when

$x = 0$, $y = \frac{2(0)-1}{3} = -\frac{1}{3}$

$x = 1$, $y = \frac{2(1)-1}{3} = \frac{1}{3}$

$x = 2$, $y = \frac{2(2)-1}{3} = 1$

$x = 3$, $y = \frac{2(3)-1}{3} = \frac{5}{3}$

$x = 4$, $y = \frac{2(4)-1}{3} = \frac{7}{3}$

$x = 5$, $y = \frac{2(5)-1}{3} = 3$

Then the range is $\left\{-\frac{1}{3},\frac{1}{3},1,\frac{5}{3},\frac{7}{3},3\right\}$.

7. $f(x) = 2x$; $f(2)$, $f(0)$, $f(-1)$
$f(2) = 2(2) = 4$, $f(0) = 2(0) = 0$, $f(-1) = 2(-1) = -2$

12. $f(s) = 5s + 2$; $f(-2)$, $f(3)$, $f\left(-\frac{2}{5}\right)$
$f(-2) = 5(-2) + 2 = -10 + 2 = -8$
$f(3) = 5(3) + 2 = 15 + 2 = 17$
$f\left(-\frac{2}{5}\right) = 5\left(-\frac{2}{5}\right) + 2 = -2 + 2 = 0$

13. $f(x) = 3$; $f(-7)$, $f(0)$, $f(0.41)$
$f(-7) = 3$, $f(0) = 3$, $f(0.41) = 3$

16. $f(x) = 7 - 2x$; $f(-4)$, $f\left(-\frac{1}{2}\right)$, $f(0)$, $f\left(\frac{1}{2}\right)$, $f(1.7)$
$f(-4) = 7 - 2(-4) = 7 + 8 = 15$ $(-4,15)$
$f\left(-\frac{1}{2}\right) = 7 - 2\left(-\frac{1}{2}\right) = 7 + 1 = 8$ $\left(-\frac{1}{2},8\right)$
$f(0) = 7 - 2(0) = 7 - 0 = 7$ $(0,7)$
$f\left(\frac{1}{2}\right) = 7 - 2\left(\frac{1}{2}\right) = 7 - 1 = 6$ $\left(\frac{1}{2},6\right)$
$f(1.7) = 7 - 2(1.7) = 7 - 3.4 = 3.6$ $(1.7,3.6)$

21. $f(C) = \frac{9}{5}C + 32$

$f(100) = \frac{9}{5}(100) + 32 = 9(20) + 32 = 180 + 32 = 212$

$f(0) = \frac{9}{5}(0) + 32 = 0 + 32 = 32$

$f(25) = \frac{9}{5}(25) + 32 = 9(5) + 32 = 45 + 32 = 77$

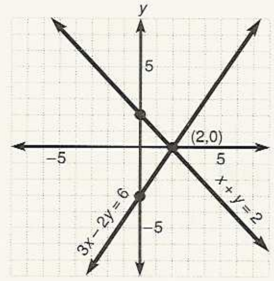
$f(-10) = \frac{9}{5}(-10) + 32 = 9(-2) + 32 = -18 + 32 = 14$

26. Using $F_f = \mu N$, where $\mu = 0.5$, $F_f = 0.5N$ and we define function h by $h(N) = 0.5N$. Then
$h(100) = 0.5(100) = 50$
$h(50) = 0.5(50) = 25$
$h(25) = 0.5(25) = 12.5$

Review exercises

1. $y = -2x + 7$ **2.** $y = \frac{3x-8}{4}$ **3.** $y = \frac{3}{5}$

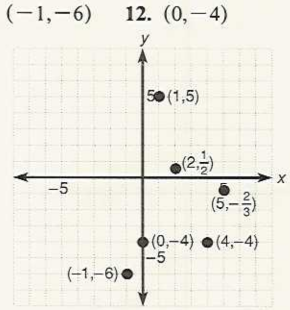
4. Point of intersection is $(2,0)$.



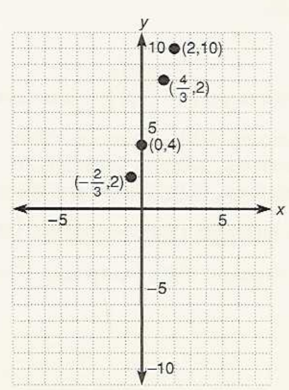
5. $x = 10$ **6.** $y = 2$

Chapter 7 review

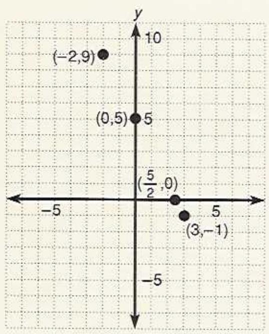
1. $(-1,1)$, $(0,4)$, $(4,16)$ **2.** $(-2,-1)$, $\left(0,\frac{1}{3}\right)$, $(1,1)$
3. $(-7,-3)$, $(0,-3)$, $(5,-3)$ **4.** $(-3,15)$, $(0,0)$, $(3,-15)$
5. $(7,-2)$, $(1,0)$, $(-14,5)$ **6.** $\left(\frac{9}{4},-1\right)$, $\left(\frac{7}{4},0\right)$, $\left(\frac{1}{4},3\right)$
7. $(1,-8)$, $(1,0)$, $(1,2)$ **8.** $(-2,-3)$, $(0,0)$, $\left(\frac{2}{3},1\right)$
9. $(1,5)$ **10.** $(4,-4)$ **11.** $(-1,-6)$ **12.** $(0,-4)$
13. $\left(2,\frac{1}{2}\right)$ **14.** $\left(5,-\frac{2}{3}\right)$



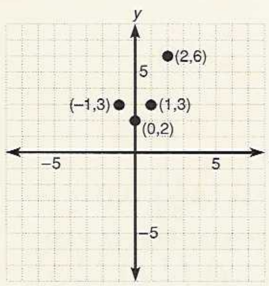
15. $(-1,-2)$ **16.** $(1,1)$ **17.** $(5,1)$ **18.** $(0,3)$ **19.** $(-2,1)$
20. $(3,-4)$ **21.** $y = 3x + 4$; $(2,10)$, $(0,4)$, $\left(\frac{4}{3},8\right)$, $\left(-\frac{2}{3},2\right)$



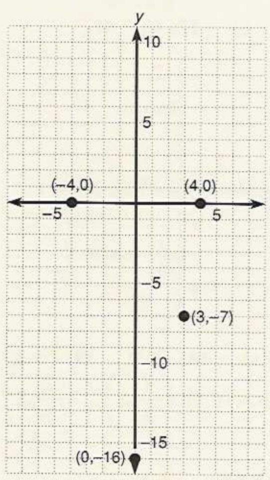
22. $y = -2x + 5$; $(-2,9)$, $\left(\frac{5}{2},0\right)$, $(0,5)$, $(3,-1)$



23. $y = x^2 + 2$; $(1,3)$, $(-1,3)$, $(0,2)$, $(2,6)$



24. $y = x^2 - 16$; $(4,0)$, $(3,-7)$, $(0,-16)$, $(-4,0)$



25. $y = 3x + 5$; x-intercept, $\left(-\frac{5}{3},0\right)$; y-intercept, $(0,5)$

26. $y = -4x$; x-intercept, $(0,0)$; y-intercept, $(0,0)$

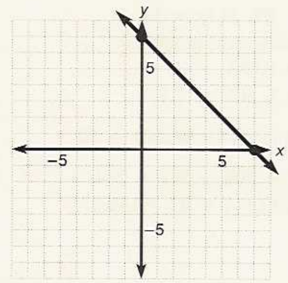
27. $y + 2 = 0$; x-intercept, (none); y-intercept, $(0,-2)$

28. $x - 6 = 0$; x-intercept, $(6,0)$; y-intercept, (none)

29. $2x - 7y = 4$; x-intercept, $(2,0)$; y-intercept, $\left(0,-\frac{4}{7}\right)$

30. $4y - x = 0$; x-intercept, $(0,0)$; y-intercept, $(0,0)$

31.



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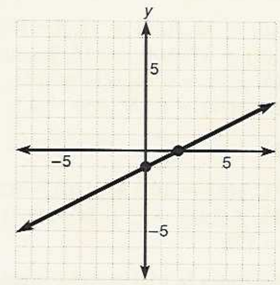

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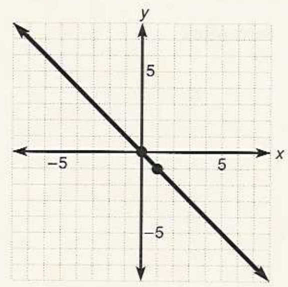

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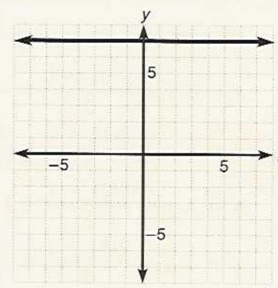

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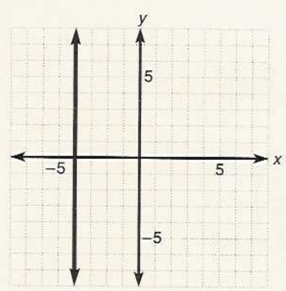

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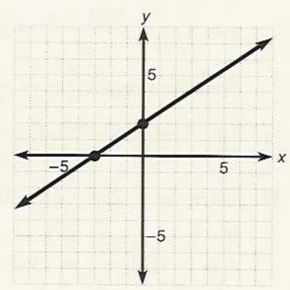

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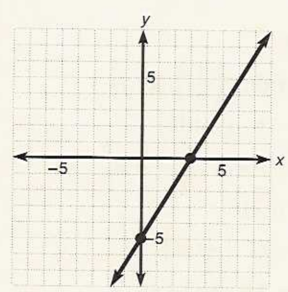
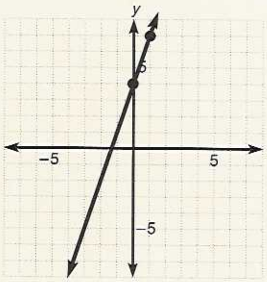


38. $(-4,3)$ and $(1,0)$; $m = -\frac{3}{5}$ **39.** $(-5,3)$ and $(-5,-1)$; $m =$ undefined **40.** $(1,-2)$, $(5,-2)$; $m = \frac{0}{4} = 0$

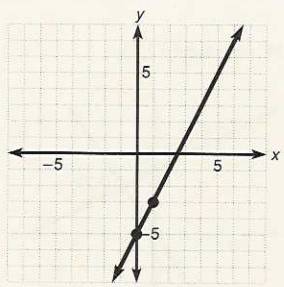
41. $(-4,-4)$, $(1,1)$; $m = 1$ **42.** $y = \frac{3}{4}x - 2$; $m = \frac{3}{4}$; $b = -2$

43. $y = -\frac{4}{3}x + \frac{2}{3}$; $m = -\frac{4}{3}$; $b = \frac{2}{3}$

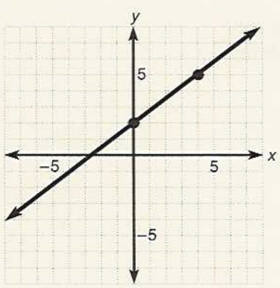
44. $y = \frac{8}{3}x + \frac{1}{3}$; $m = \frac{8}{3}$; $b = \frac{1}{3}$ **45.** $y = 5x - 10$; $m = 5$; $b = -10$
46. $m = -1$ passing through $(0,-1)$; $y + x = -1$
47. $m = 1$ passing through $(2,0)$; $y - x = -2$
48. $m = \frac{150}{1{,}050} = \frac{3}{21} = \frac{1}{7}$ **49.** $y + 4x = 21$
50. $-x + 3y = 12$
51. $y - 2 = \frac{1}{8}(x - 5)$; $8y - 16 = x - 5$; $x - 8y = -11$
52. $-4x + 3y = 12$
53. $m = 3$; $b = 4$



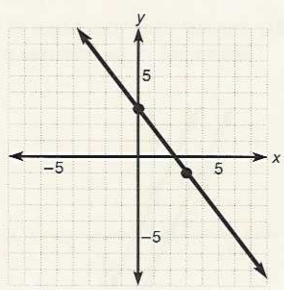
54. $m = 2$; $b = -5$



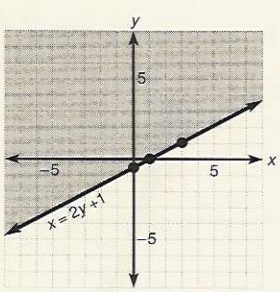
55. $m = \frac{3}{4}$; $b = 2$



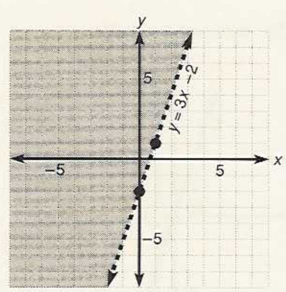
56. $m = -\frac{4}{3}$; $b = 3$



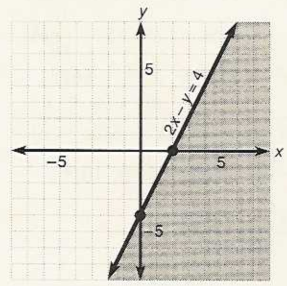
57. $x \le 2y + 1$



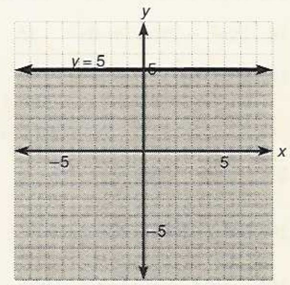
58. $y > 3x - 2$



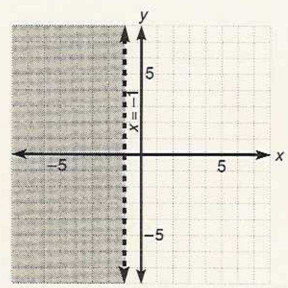
59. $2x - y \ge 4$



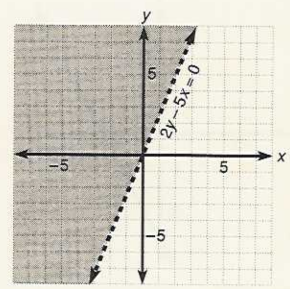
60. $y \le 5$



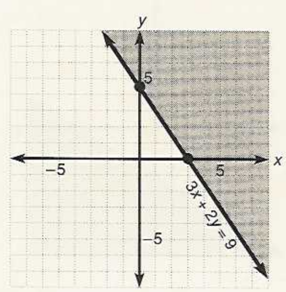
61. $x < -1$



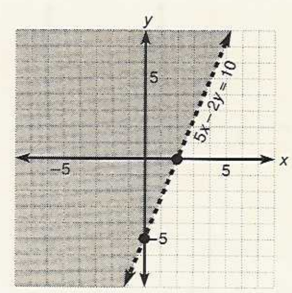
62. $2y - 5x > 0$



63. $3x + 2y \ge 9$



64. $5x - 2y < 10$

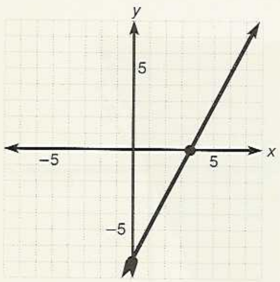


65. domain $= \{-5,-3,-1,0,1,3,5\}$, range $= \{14,8,2,-1,-4,-10,-16\}$
66. domain $= \{-4,-1,0,2,3\}$, range $= \left\{\frac{7}{4},1,\frac{3}{4},\frac{1}{4},0\right\}$
67. domain $= \{-7,-3,0,2,6\}$, range $= \left\{\frac{-7}{2},\frac{-3}{2},0,1,3\right\}$
68. $f(-1) = -4, f(0) = 0, f(4) = 16$; $(-1,-4)$, $(0,0)$, $(4,16)$
69. $f(-2) = 6, f(0) = 4, f(2) = 2$; $(-2,6)$, $(0,4)$, $(2,2)$
70. $f(-4) = -23, f(0) = -3, f(6) = 27$; $(-4,-23)$, $(0,-3)$, $(6,27)$ **71.** $f(-8) = 5, f(0) = 5, f(9) = 5$; $(-8,5)$, $(0,5)$, $(9,5)$
72. $f(s) = 6s$, $f(3) = 18$, $f(14) = 84$; domain $=$ {positive real numbers}

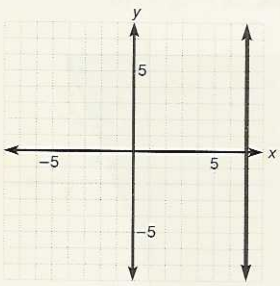
Chapter 7 cumulative test

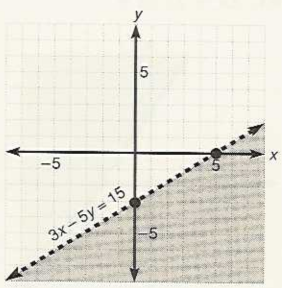
1. 25 **2.** $-27x + 36$ **3.** $\left\{\frac{8}{3}\right\}$ **4.** $\left\{\frac{60}{17}\right\}$ **5.** $\left\{-\frac{5}{3}\right\}$
6. $\left\{-\frac{2}{3},2\right\}$ **7.** $-1 < x \le 4$ **8.** $(7x + 1)(x - 1)$

9. $y^6(y^2 + y - 1)$ **10.** $p(p^3 - 4)(p + 1)$
11. $16(a + b)(a - b)$ **12.** $t(t - 7)(t - 1)$ **13.** $-7(5y + 4z)$
14. $16y^2 - 24xy + 9x^2$ **15.** $20x^2 - 3x - 56$ **16.** $25 - \frac{9}{16}y^2$
17. $x^3 - 3x^2y + 3xy^2 - y^3$ **18.** $15y^5 + 18y^4 + 32y^3 + 66y^2 - 7y + 56$ **19.** $\frac{5x^2 + 32x}{(x - 7)(x + 7)(x + 6)}$ **20.** $\frac{a^2 - 2a + 1}{(a + 5)(a - 2)}$
21. $\frac{6}{y + 3}$ **22.** $\frac{1}{x^9}$ **23.** $\frac{1}{8^8}$ **24.** $-\frac{125}{a^9}$ **25.** 7.76×10^{-6}
26. $3x^2 - 8x + 20 + \frac{-45}{x + 2}$ **27.** $\frac{2x^2 - x + 9}{3x + 4}$
28. x-intercept, $\frac{7}{2}$; y-intercept, -7



29. x-intercept, 7; y-intercept, none **30.**





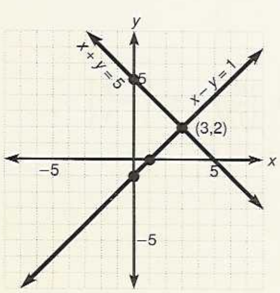
31. $m = \frac{7}{3}$ **32.** $3x - 4y = -13$ **33.** $3x + 5y = 27$
34. parallel **35.** neither **36.** $f(0) = -2, f(-3) = -11, f(4) = 10$ **37.** 136 **38.** 12 **39.** legs are 5 inches and 12 inches

Chapter 8

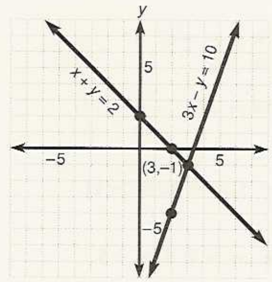
Exercise 8–1

Answers to odd-numbered problems

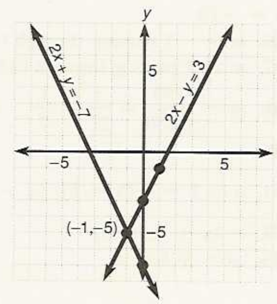
1. yes **3.** yes **5.** yes **7.** no **9.** yes **11.** no
13. solution: (3,2)



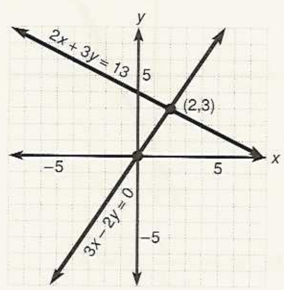
15. solution: $(3, -1)$



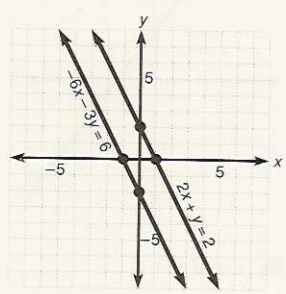
17. solution: $(-1, -5)$



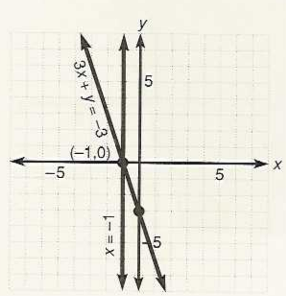
19. solution: (2,3)



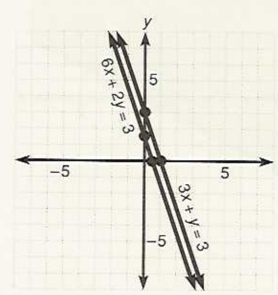
21. parallel lines (the slopes of both lines are the same) ∴ *inconsistent* with no solution



23. solution: $(-1, 0)$



25. inconsistent; no solution



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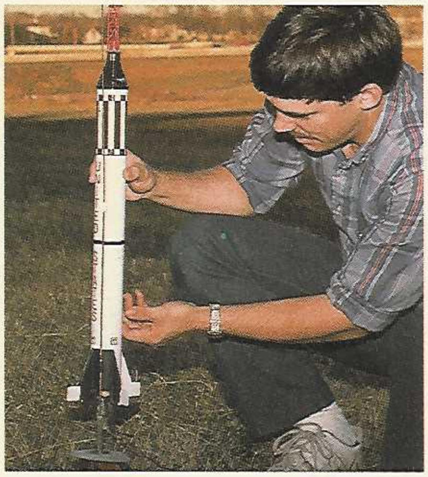

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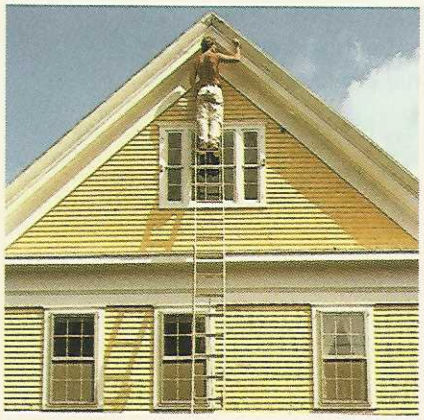
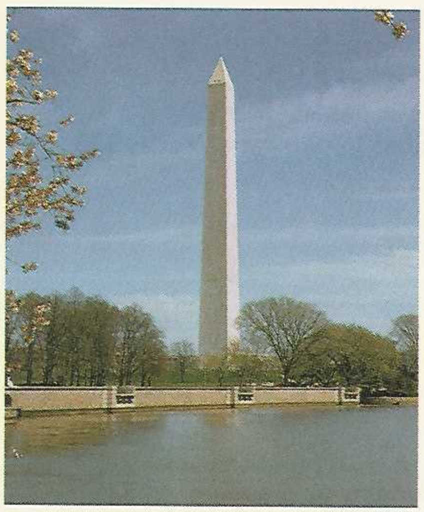
Chapter 8 ■ Systems of Linear Equations

Chapter 9 ■ Roots and Radicals



Chapter 10 ■ Solutions of Quadratic Equations



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